LEONARDO DA VINCI

THE VITRUVIAN MAN

THE DESIGN AND THE GOLDEN SECTION

FORMAL ANALYSIS
AND
DIDACTICAL GEOMETRIC RECONSTRUCTION
Leonardo da Vinci’s autographic drawing about the canon of the human figure proportions known as the "Vitruvian Man", as well as the "Uomo di Venezia", is today the best known images among the master’s thousands of drawings that have survived to reach us.

Thanks to its frequent reproduction as well as its having been selected as our European currency symbol, the image has been even more widely disseminated and subjected to more interpretations than the Gioconda.

The Gioconda and the Vitruvian Man, peaks of excellence in their respective media (painting and drawing) have turned Leonardo into the very icon of that humanism that evolved into the Renaissance.

One may certainly argue that universal admiration for Leonardo arose with his earliest productions.

Yet the "fortune" of these two works – in terms of how they have been studied and disseminated as well as the study and dissemination of other works directly or indirectly inspired by them – has not been synchronic.

That of the Gioconda, recognized as a masterpiece even before its completion and a source of inspiration for many artists beginning with Raphael, rose to heights earlier.

The fortune of the drawing, on the other hand, rose more recently, in our own period. Despite attracting frequent and considerable attention earlier in history, its fate was essentially sealed in 1949 by R. Wittkower’s ARCHITECTURAL PRINCIPLES IN THE AGE OF HUMANISM (New York, Norton and Co., 1971)2.

Indeed, its fame is of such recent origin that its significance has been trivialized in a typically modern manner.

Often it has been understood as a mere drawing exercise, and as such, adapted to a wide range of fanciful uses in the most banal manner.

There is nothing wrong with this. No one was shocked by a Gioconda with a moustache any more than we are shocked today by an erotic version of the Vitruvian Man – all the less so by the synthesized one used as the mascot for the Milan Expo.

Nothing wrong as long as simplified derivations of this sort are never used to study the original work which, unique in its complexity and profound in its philosophical motives, was the resulting product of research, of the study of nature and anatomy, as well as of refined and precise execution. Indeed even the slightest simplification could lead to false readings.

The Vitruvian Man is an absolute masterpiece. It bears the name of Vitruvius and reveals the inspiration of his text on the insertion of man in a circle and square. 3

See as follows Leonardo’s interpretation 1) of Vitruvius’ words compared with the interpretations of Francesco di Giorgio 2), Cesare Cesariano 3) and Fra Giocondo 4).

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1 Sheet dimension about mm 344x245. Pen refinished metal tip, ink and watercolour. Galleria dell’Accademia, inv. 228 Venezia.

2 In his recent LEONARDO & IO, Carlo Pedretti notes that the earliest reproductions of the Vitruvian Man appear in Gerli (1784) and Bossi (1810) as well as in the learned disputes that followed, which were limited to a specialized public. Different and surprising in its scope was the attention -- even polemical -- raised by the aforesaid work by R. Wittcower.

3 Marcus Vitruvius Pollio (80/70 - 23 B.C.E.) wrote his treatise De Architectura in ten books and dedicated it to Augustus. Rediscovered and translated in the Renaissance (1414) by Poggio Bracciolini, it lies at the basis of Leon Battista Albert’s DE RE AEDIFICATORIA.

In Bk. III, cap. I of De Architectura Vitruvius writes: 

Item corporis centrum medium naturaliter est umbilicus. Namque si homo conlocatus fuerit supinus manibus et pedibus pannis circcinique conlocatum centrum in umbilico eius, circumagendo rotundationem utramque manus et pedum digitii linea tangentur. Non minus quamadmodum schema rotundationis in corpore efficitur, item quadrata designatio in eo inventetur. Nam si a pedibus imis ad summum caput mensum erit eaque mensura relata fuerit ad manus pannis, inventetur eadem latitudo ati altitudo, quamadmodum areae, quae ad normam sunt quadratue.
Several artists between the early fifteenth century and the mid-sixteenth century attempted a humanist interpretations or graphic renderings of Vitruvius' words. Of these, Leonardo's solely still fascinates today.

Perhaps this is also because, apart from the elegance and the beauty of the drawing, we, in our present sensibility, appreciate the superimposition of the man's two different positions in the circle and square as a movement captured through sequential modern freeze frames.

Above any other consideration, this drawing differs from all other interpretations of the Vitruvian text because Leonardo superimposes, to its literal meaning, the humanist sentiment of the period as well as his own personal and profound comprehension of the laws of nature. Thus, the drawing blends in one extraordinary scheme two figures: the one inserted in the square with open arms and joined legs and the other perfectly inscribed in a circle with arms raised and legs splayed.

Typical of Leonardo is the drawing's beauty, which grants visual form to anatomical perfection down to the detail of the right leg of the man inserted in the square. His right foot lies orthogonally to the figure, bearing the body's weight in a natural manner. His left, disengaged leg, is instead parallel to the plane of the body and prepares to open with a movement, which along with that of the arm, will carry it up, thereby concurring through its impetus, which naturally releases it, to the right leg's simultaneous movement.

Furthermore, Leonardo, in this Work, introduces also a new, unheard of and fascinating interpretation of Piero della Francesca's great lesson. In fact, while in the great forerunner's paintings the geometric constructions organizing the relationships between the represented figures are just implied, in the Vitruvian Man these constructions become the protagonists of the Work and the interpreters of the laws with which nature proportions the human figure, frames it and exalts it as a microcosm that in itself reassumes all those laws.

Laws that Leonardo has day by day perused in all fields, from animal and human anatomy, to plants and flowers study, to the analysis of the forces acting in the constructions, in the mechanical and the optical realms. Inquiries conducted with most accurate proportions measurements and often supported by geometric syntheses (see more ahead to page 6 and notes 12 and 13) analogous to the ones of this Drawing.

In such a context one cannot avoid noticing that it exists a precise geometrical relationship between the circumference circumscribing the figure of the man with raised arms and spread leg, and the square in which it fits when drawn with united legs and opened arms. Relationship that exists independently from and unrelated to the one organically binding the two overlapping figures.

Relationship that must exist not only because of the sought after and evident tangency between the circumference and the square base side but also because of the unquestionable graphical perfection that pervades and defines the Sheet's composition in all of its part, drawn and written, and that manifestly originates precisely from the design of the circumference and of the square.

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1 The humanist concept of man, as the center and measure of all things, found its confirmation in a particular, possibly biased, interpretation of Vitruvius' words, drawing from this interpretation inspiration for rendering a man inscribed in the circle and square.

In the square as in the circle proportioned to the figure of a man, Romans read pure and simple ways of adjusting their practical units of measurement: rods, paces, arm-lengths, etc. The objectives of Renaissance artists naturally differed. Man, understood as the microcosm at the center of the universe, represented by a circle, formally and symbolically constituted the real and tangible unity of the measure through his inscription in the square, in accordance with the Vitruvius's text and intention.

2 In the Sheet the drafted part and the written parts, as well as the parts left empty between drawing and text and to the margins, have been composed in spaces geometrically defined according to the drawing of the circle and the square. This is not only suggested from the elegant adaptation of the written part to the drafted one but can be easily checked sliding over the Sheet an other one of transparent paper on which have been traced the drawing of the circumference and the square with the
One can therefore see that circumference and square are related in a way that is independent from the biform figure that they frame.

It exists therefore, in the Drawing, another hidden geometric construction that is subtended to the one of the circumference and the square between them tangent.

Leonardo himself makes this understood in the text in which he lists the measurements and resumes the proportions of the Vitruvian Man. There is, in fact, a fundamental passage where he specifies that the circumference centre, circumscribed to the figure with raised arms and spread legs, will coincide with the navel if the space opened between the legs will be congruent with the one of an equilateral triangle.6

Finally, as we will see later in detail, one can recognize in the arms' slant as in the legs' splay, the various steps that, leading to the design of a regular pentagon, complete this construction.

Steps in which one can recognize implicit references to the studies conducted together with Luca Pacioli 7 in the drawing up of DIVINE DE PROPORTEONE. However, while the presence of the equilateral triangle is explicit, the design of the pentagon stays implied and its construction is only suggested (one could say in subliminal way) as logically it must be, in the spirit of the time, being a matter of a construction executed to design the dodecahedron, the secret "quintessence".8

Yet, the construction tying together circle and square, for as much as it can be easily and intuitively perceived, so it appears as well to elude every detailed geometrical and/or mathematical setup.

Recently someone has even claimed to have identified it in a direct relationship9. The radius of the circle would be equivalent to the golden section of the square's side. Actually, this could be so if only we were willing to accept a rough approximation. Therefore it is not. Because there can be no approximation in Leonardo's drawings.

Then, if we agree with Ragghianti's argument that visual arts have not only aesthetic but also cognitive purport – and this is certainly true of all of Leonardo's works, and to an even higher degree of this one – then our study must be all the more attentive, precise, and correct.

To understand really the value of the work, it is just not enough to guess that Leonardo da Vinci implies the Golden Section in the geometrical construction of the Vitruvian Man "to have it demonstrated".

According to Paolo Alberto Rossi, to understand is to preserve – in the proper sense of preserving value10. One must then remember with him what Plato wished to have inscribed over the door of his Academy, namely, that those ignorant of geometry were not to enter: and "that geometric calculations - always in the platonic teaching - are the knowledge not the execution."11.

circle fitting into the square, the regular pentagon in it inscribed and the equilateral triangle mentioned by Leonardo (See ahead - Analytical Reconstruction).

1 For the passage in which Leonardo explains how a standing man inscribed in a square can be transformed into a homo ad circulum; see note 18. Regarding the relationship among square, equilateral triangle and pentagon see fig. 2 at the next page.

2 Luca Pacioli, a mathematician from San Sepolcro, just a few years younger than Leonardo, was called to the court of Ludovico il Moro most probably at the suggestion of Leonardo himself since he had already published and dedicated his SUMMA DE ARITMETICA, GEOMETRIA, PROPORTIONI E PROPORIONALITÀ to Guidobaldo, the Duke of Urbino. At this point, he had already enjoyed a prestigious career as teacher, scholar, and disseminator of the works of Piero della Francesca, his illustrious fellow citizen, and Leon Battista Alberti. His encounter with Leonardo in 1496 and 1500 led to close association, collaboration, and reciprocal esteem, and his most famous work, DE DIVINA PROPORTEONE, was illustrated with sixty drawings by the artist's "ineffabile sinistra mano" ("awesome left hand" in the words of Pacioli himself).

3 In nature there exist only five regular polyhedrons such that their sides are constituted by regular polygons equal among themselves: the cube, tetrahedron, octahedron, icosahedron, and dodecahedron. In DE DIVINA PROPORTEONE, Luca Pacioli refers to the link that Plato advances in his Timaeus between the first four polyhedrons and the four Empedoclean elements – earth, fire, air, and water respectively. To the dodecahedron, constituted of twelve pentagon sides, is assigned the Fifth-Essence of Greek physics, which, according to neo-Platonic philosophy, is understood as the true and proper soul of the world. This is because the regular pentagon can be drawn only with the knowledge of the construction of the Golden Section of a segment. The Golden Section of segment a is that which divides it in two parts a and b, such that a:b = a:b. This ratio, defined by Pacioli as the divine proportion, possibly following a suggestion of Leonardo himself, is the basis and rule of countless phenomena of natural growth.

4 Rocco Sinisgalli, LEONARDO PER GIRO, L'UOMO VITRUVIANO, Federighi Editor, Firenze 2006.

5 See Paolo Alberto Rossi, LE CUPOLE DEL BRUNELLESCHI, Bologna: Edizioni Calderini, 1982

6 Paolo Alberto Rossi, ib. Page. 3.
Thus follows this geometrical analysis of the *Vitruvian Man*, through which we can prove that the Golden Ratio permeates the entire construction and discloses the rule of the body's proportion as identified by Leonardo's genius. The drawing conveys the true and proper laws of nature, which determine its proportions through graphic means, though in a subtle and cryptic manner appropriate to Art, particularly Leonardo's Art. The ray of the circle, in which the human figure is inscribed, is a function of the square's side, the truth of which will become perfectly clear in an articulated and complex way, in a point-by-point description of the geometric construction.

What follows is a reconstruction based on clues present in the drawing itself, but of which I became aware only after having seen a drawing in which Leonardo (1)&(2) depicted a flower's evolution from five to six petals[^12^], and another by Carlo Urbino (3), deemed by most scholars a copy of a Leonardo's lost drawing[^13^].

![Image](image.jpg)

Evident in both the above-mentioned drawings is the search for a geometrical link between the equilateral triangle and the pentagon. In the drawing, 1, Leonardo does not trace the equilateral triangle but the way to make it and writes under the drawing "*sia data la linea ab farai il triangolo equilatero adb*" (*given the ab line you will make the equilateral triangle adb*). Then he draws the regular pentagon having side ab and explains that, in order to construct it, after dividing the height dc of the equilateral triangle in 5 equal parts, the circumference that circumscribes it has centre in the fourth part of this height. The geometrical construction that allows moving from the equilateral triangle to the pentagon, albeit approximate, preludes in an amazing way to the Drawing of the Vitruvian Man geometry.

This I have evidenced superimposing, 2, to the Leonardo's drawing the drawing of a square circumscribed to the circumference that circumscribes the pentagon. Thereafter, in red ink, I have added the equilateral triangle, mentioned by Leonardo in the Vitruvian Man text, along with, always in red, the circumference, tangent to the base side of the square circumscribed to the circle in which the pentagon fits having centre in the point d, apex of the equilateral triangle.

In Carlo Urbino’s drawing (3), the link is glimpsed solely among the many geometrical deformations and fantasies that pervade the drawing and that lead one to believe, as Giulio Bora claims, that the painter from Cremona had seen other preparatory drawings by Leonardo for this one of the Vitruvian Man.[^14^]

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[^12^]: Foglio 14r Ms. B. 2173, Paris Bibliothéque de France, (pen and china on paper mm 233x168), see page 43 as well as note 34 of my study: **LEONARDO, GLI SCACCHI** .


[^14^]: Giulio Bora, **LEONARDO E LA COSTRUZIONE DEI CORPI**, as cited in n. 7.
THE VITRUVIAN MAN
A DIDACTICAL GEOMETRIC RECONSTRUCTION

Numerous visual elements suggest that Leonardo used the Golden Section to construct his drawing. Yet, as of today, no one – as far as I know – has demonstrated this.

In particular, many have claimed that the navel at the center of the circle that circumscribes the man with raised arms and splayed legs divides the height of the standing man by the extreme and mean ratio. This is not so as the construction below clearly demonstrates. Indeed the Golden Section of the square's side does not coincide with the height of the navel. In the original drawing, the distance between their lengths is 2 mm – an unacceptable approximation.

Fig. 1 An erroneous construction hypothesis

Yet the link between the circle and square exists and passes right through the construction of a Golden Section.

It is Leonardo himself, who, in the marginal text to the drawing, provides the first clear indication.

There he explains how a standing figure of a man inscribed in a square can be transformed into a man inscribed in a circle. In fact, he says, his arms raised to the height of his head and the legs splayed open, his height is reduced by 1/14.

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15See Professor Rocco Sinigaglia, who in LEONARDO PER GIOCO, L'UOMO VITRUVIANO (as cited in n. 9), hypothesizes that the ray of the circumscribing circle is equal to the Golden Section of the side of the square. The geometric construction hypothesized by him is conveyed here over a copy of the original drawing (Fig. 1), and is revealed as incorrect.

16The following definition appears in Euclid's Elements: "It is said that a segment is divided according to the median, and extreme ratio, when a segment is to the greater part as the greater part is to the lesser one." 

17The Golden Ratio was probably first defined by Euclid (as the ratio of the hypotenuse to one of the catheti of a right triangle) and then by Plato (as the ratio of the sides of the isosceles right triangle). The first clear indication of the equivalence of these definitions is that given by Vitruvio in his treatise De architectura.

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The diameter of the circle and the side of the square, the height of the navel, and its measurement has been carried on the square's base side has been constructed.

The segment has been transferred vertically and its measurement has been carried on the square's median.

The Golden Section of the square's side does not coincide with the height of the navel. The difference is of 12 mm 159.75.

The difference is of mm 240.75.

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\[ \Phi = \frac{\sqrt{5} - 1}{2} \]

The Golden Ratio \( a/b \), was designated with the letter \( \Phi \) for the first time by mathematicians Mark Barr and William Schooling; Theodore Andrea Cook, The Curves of Life (Dover Books, Explaining Science). The value of \( \Phi \) is determined by expanding the two given equations: \( \Phi = a/b = b/c \), \( b/a = a/\Phi \). The quadratic equation \( \Phi^2 - \Phi - 1 = 0 \) for \( a = 1 \) drawn from \( \Phi = \sqrt{5}/2 + 1/2 = 1.618033988 \) is the Golden Ratio, as defined above. Its inverse, \( 1/\Phi = 0.618033988 \) is the Golden Section, that is, part of segment AB.

In the geometric construction outlined above, if the side of the square is 1, then – because according to Pythagoras' theorem \( \sqrt{5}/2 \) is the measure of the diagonal of the semi-square of side 1 – it is possible to construct the Golden Section of a given unit by subtracting from it \( \sqrt{5}/2 \), the diagonal of the semi-square of side 1, half the side, precisely 1/2. In particular, \( \sqrt{5}/2 - 1/2 \) is the length of the side of the decagon inscribed in the circumference with a radius equal to 1.

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and the open legs form an equilateral triangle whose top vertex coincides with the navel, which is at the center of the circle in which he is inscribed. Fig. 2. 18.

The first clue is embedded in Leonardo's studies of the development of five- and six-petal flowers, in which he emphasizes the link between an equilateral triangle and a pentagon (see p. 6).

Fig. 2

Fig. 3

Fig. 4

Fig. 5

The second clue becomes clear indeed when a circle, which we call C' in order to distinguish it from the circle drawn by Leonardo, which we call C, is inscribed into the square drawn by Leonardo, which we call Q. Fig. 3.

In fact, if we draw a regular pentagon into this circle C', we can see what happens to the sides of the equilateral triangle mentioned by Leonardo, namely, that the raised arms of the man inscribed in circle C intersect C' at exactly the same point of this regular pentagon's sides. Pls. 4-5.

We also note that Leonardo draws two points at the center of the raised arms' forearm, points that are insignificant from an anatomical point of view, but important because they single out the intersection between the C' circle and the pentagon's side Fig. 5.

The third clue that helps reconstruct the geometric and analytical bond between the side of square Q and the radius R of circle C lies in the slant of the axis of the raised arm that runs parallel to the diagonal of the semi-square constructed over the vertical radius of C.

18Fourth, fifth and sixth line in the text: "se tu apri tanto le gambe che tu cali da capo 1/14 di tua altezza, e apri e alza le braccia che colle lunghe dita tu tochi la linea della sommità del capo, sappi che l' centro delle stremità delle aperte membra fia il bellico, e llo spazio che ssi truova infra lle gambe fia triangolo equilatero": "if you so much open the legs so that you reduce from your head a 1/14 of your height, and open and raise your arms so that with your long fingers you touch the line of the top of the head, be aware that the centre of the extremities of the opened limbs will be the belly button and the space existing between the legs will make an equilateral triangle". An equilateral triangle that has the superior apex in the navel and for base the side of the pentagon fitting the circle circumscribed by the square. Leonardo cites the equilateral triangle but holds hidden the reference to the pentagon.
In effect this diagonal $D$ is tied to the radius $R$ of the circle $C$ by the simple Pythagoras's relationship $D = R\sqrt{5}/2$. On the other hand we also know that the length of the side $d$ of the decagon inscribed in that very same circle of radius $R$ is tied to the latter by the relationship $d = R(\sqrt{5}-1)/2$.

![Fig. 6](image6.png)  ![Fig. 7](image7.png)

We can thus inscribe into the circle $C$ a decagon of side $d$ just by subtracting half of the radius $R$ from that diagonal $D$ whose slant is given by the Vitruvian man raised arm, see note 15. Fig. 6.

Now this is just the very first step to the road designing the geometric construction that binds univocally the circle $C$ and the square $Q$ demonstrating that the side of the latter is a function of the radius of the former.

We note, in fact, that, this decagon intersects the sides of the equilateral triangle mentioned by Leonardo in two points, see (Fig. 7) and that by construction, the segment joining them is the side of the regular pentagon inscribed in a circle, that we shall call $C'$, passing through these two points and the decagon's vertex. Pls. 8 and 9.

![Fig. 8](image8.png)  ![Fig. 9](image9.png)

Such circle $C'$ results inscribed with great precision in the $Q$ square, as evidenced in the geometric construction traced over one copy of Leonardo's drawing.
Nevertheless, the circle $C'$ as well as the pentagon in its inscribed are implied but not to be seen in Leonardo's drawing. Thus circle $C'$ is not represented but exists within the geometric construction that was sketched and then removed, that is deleted, by Leonardo.

However, while of the pentagon drawing there is no trace left, the drawing of this circle must have left a clear sign in the centre of the square where in order to trace it Leonardo put his compasses.

In conclusion, the geometric, one-to-one bond that unites circle $C$ to side $L$ of square $Q$ can be expressed synthetically as follows:

The circle $C$ having radius $R$ and the square $Q$ having side $L$ must be tangent (at the bottom of the square) in such a way that the two radii $R$, side of the equilateral triangle inscribed in $C$ having vertex in the center of $C$, cut on the circle $C'$ inscribed in the square $Q$ a segment equal to the side of the regular pentagon inscribed in $C'$.

Possibly, the construction could have followed the reverse course, and be traced from the side of the square in which is inscribed the man standing with joined legs and open arms to the circle encompassing the man with splayed legs and raised arms. The sequence would then be: We inscribe a circle $C'$ in square $Q$ (Fig. 10). Then we construct a regular pentagon inscribed in $C'$. (Fig. 11)

![Fig. 10](image1)
![Fig. 11](image2)
![Fig. 12](image3)

Finally, on the base of this regular pentagon we construct an equilateral triangle (Fig. 12), whose upper vertex coincides with the figure's navel and the center of circle $C$ circumscribing the man with raised arms and splayed legs. In such a way, as shown in the construction of Fig. 12, laid upon Leonardo's drawing, proceeding from the square we obtain a circle that coincides almost perfectly with the one designed from Leonardo.

For this geometric reconstruction of mine as well, we can calculate analytically, the measurement of $R$ in function of $L$.

Thus we can measure the difference between the calculated value of $2R$, and the diameter's measurement of the circumference circumscribed to the figure of the Man with arms raised the legs splayed as found, on Leonardo's drawing.

As follows Fig. 13:

Fig.13.1: Let's draw the circumference $C'$, inscribed in the square $Q$ having side $L$, and the regular pentagon $P$, inscribed in $C'$

Fig.13.2: On a $d'd'$ side of $P$ let's draw the equilateral triangle $d'd'f$ inside $C'$

Let's trace the diameter of $C'$ passing through $f$ and through the central point $h$ of the side $d'd'$. Let's call $a$ and $b$ the points where this diameter cuts $C'$

Fig.13.3: Our unknown is the radius $R$ of circumference $C$ tangent to the circumference $C'$ having for centre the vertex $f$ of the equilateral triangle constructed, inside $C'$, on a side $d'd'$ of $P$.

$R = af = ah + hf$.

Now let's calculate $ah$.

Of the triangle rectangle $adh$ we know the sides $ah = L$ and $ad$, because side of the decagon inscribed in the circumference $C'$ having diameter $= L$. Therefore $ad = L*15\sqrt{3} - 1)/2$. (The side of the decagon is congruent with the golden section of the radius of the circumference that the circumscribing it, see note 16).

19The correct term here is "removed" rather than "erased". In fact, as was the case with the paper used back then, it was impossible to erase anything without blotching it. For this reason, preparatory lines for drawings that were not meant to appear in the finished work were done with a thin point, usually silverpoint, on a sheet coated with ash or talc that was later blown away, thereby destroying all trace of the preliminary sketch. If a construction had anticipated the drawing of a circle, all that would have been left of the latter, was a pinprick in the center created by the compasses' point, as was the case here. Of this circle perhaps should remain another faint trace visible in the left of the top half. Nevertheless, to be certain of this, one would need to examine the original Drawing.

20This mark, of which I indicate the position without ever having held the Sheet in my hands and without ever having heard of, will be the proof of the correctness of my analysis. Furthermore, the construction of the pentagon also must have left trace of one or more compasses pinpricks but of these I cannot indicate the position because the construction can have been made in various parts of the Sheet

The Man of Vitruvio
Always in the right-angle triangle adb for the Euclidean theorem $ah^2 = ah * ab$
Therefore $ah = \frac{ah^2}{ab} = \frac{[L \cdot (\sqrt{5} - 1/2)]^2}{2L} = \frac{0.0955L}{L}$
Let's calculate $hf$:
Of the right-angle triangle dfh for Pythagoras' theorem $hf^2 = df^2 - dh^2$;
But $df$ for construction is equal to $dd' = 2dh$.
Therefore $hf = \sqrt{(4dh^2 - dh^2)} = dh \cdot \sqrt{3}$; but $dh^2$,
For the second Euclidean's theorem applied to the right-angle triangle adb, $dh^2 = ah \cdot hb$;
But $hb = L - ah$; thus replacing $ah$ with $0.0955^2L$, $hb = L - 0.0955^2L^2$ $hb = 0.9045L$
Therefore $dh^2 = 0.0955L - 0.9045L = 0.086397L^2$;
Therefore $dh = 0.2939L$ and $hf = 0.2939L \cdot \sqrt{3}$ thus $hf = 0.5090L$
Therefore $R = ah + hf = (0.0955 + 0.5090) L = 0.6045L$
Placing $L = mm 181.475$ (see note 17 at page 7), $2R = mm 219.40$
Which related to the measure, mm 219.5 of the diameter of C
as measured on Leonardo's drawing shows a difference of mm 0