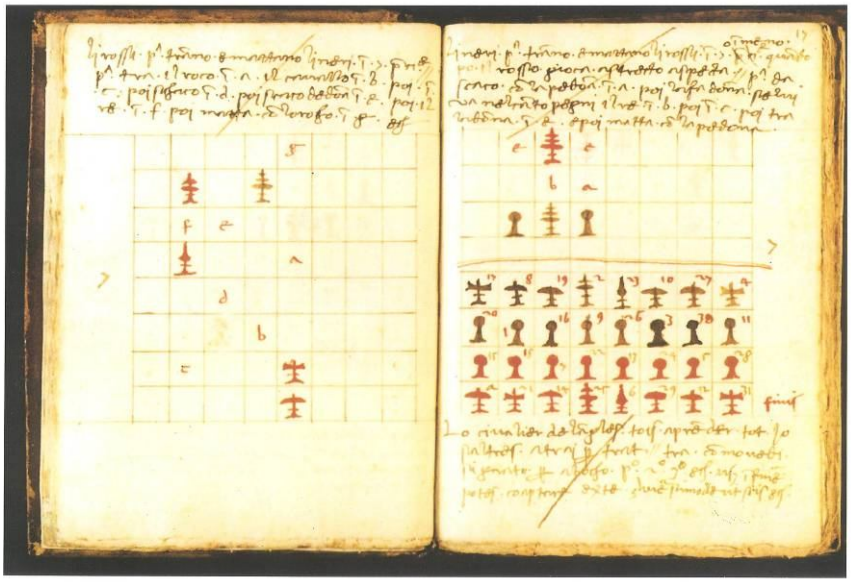
LEONARDO

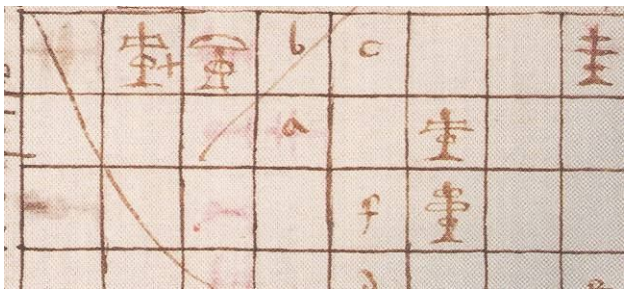
GLI SCACCHI

NEL MANOSCRITTO SUL GIOCO
RECENTEMENTE RITROVATO E RICONOSCIUTO AUTOGRAFO
DI

LUCA PACIOLI



FORMA DEI PEZZI – RAPPRESENTAZIONE GRAFICA – PROBLEMI



INDEX

FOREWORD

On December 27, 2006 Ansa reported the discovery of the autograph manuscript of DE LUDO SCHACHORUM o SHIFANOIA, a treatise on the game of chess by Luca Pacioli. Deemed lost for centuries, it was known to scholars only through Pacioli's own testimony.

Knowing not only of my interest in chess piece design,[1] but also of the references to Renaissance neo-platonic geometry, particularly Luca Pacioli's DE DIVINA PROPORTIONE in my writings,[2] Adolivio Capece, national chess master and editor of the magazine L'ITALIA SCACCHISTICA, quickly informed me of the discovery.[3]

The Ansa report referred to an event of December 20, 2006, when Dr. Serenella Ferrari Benedetti, the cultural coordinator of the Fondazione Coronini Cronberg in Gorizia, Italy, brought to the bibliophile Duilio Contin's attention a precious late fifteenth-century manuscript on the game of chess in the foundation's 22,000-volume library. Dr. Contin was at the Corinini at the request of the Società Aboca di Borgo San Sepolcro in Arezzo, which had engaged him to research Piero della Francesca (1420-1492) and Luca Pacioli (1445-1514 or 17), both of whom had been born in Borgo San Sepolcro.

Several months later, paleographic and linguistic analyses conducted by Professors Attilio Bartoli Langeli and Enzo Mattesini of the University of Perugia, confirmed that the manuscript was autograph, as well as a preparatory study for the famous treatise DE LUDO SCACHORUM, also known as SCHIFANOIA.[4] Referring to the treatise in another work, Luca Pacioli stated that he had written it with the intention of dedicating it to Isabella d'Este, wife of Francesco II Gonzaga.[5]

The Historical Context

The manuscript was prepared between the late fifteenth and early sixteenth century. A dated watermark on one of its page sets the *terminus post quem* more precisely as 1497, while a request for a printing privilege addressed to the Venetian Doge Leonardo Loredan establishes the *terminus ante quem* as December 29, 1508.

1497 and 1508 were decisive years in the intense and eventful life of Luca Pacioli.

1508 marked his return – after Milan, Florence, Pisa, and Bologna – to Venice, the city which had formed him and where he was to publish his chief works. 1497 in turn was the year in which he moved to Milan, where he had been invited by Duke Ludovico il Moro in 1496 to teach mathematics at court. The invitation may have been suggested and expedited by Leonardo Da Vinci, who, as is well known, had a great interest in mathematics and owned a copy of one of Pacioli's most famous books, SUMMA DE ARITHMETICA, GEOMETRIA, PROPORTIONI ET PROPORTIONALITÀ, dedicated to Guidobaldo, the Duke of Urbino, and published in Venice in 1494.

When the two met, Leonardo was forty four, Luca Pacioli around fifty one.[6] This was Luca Pacioli's first encounter with Leonardo, whose fame as an extraordinary painter was known in all the courts of Italy as well as through the poems of Bernardo Bellincioni. As noted, Leonardo was already familiar with Pacioli's work.

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[1] Particularly my chess set, named "Scaccomatto" ["Checkmate"], manufactured in a limited edition of 1000 copies, each of which was numbered and signed. An image of Scaccomatto was chosen to illustrate the entry "Chess" in the PICCOLA TRECCANI Italian enciclopedy. Scaccomatto can be viewed at www.francorocco.com.

[2] In "Research Pathways," *Agip Review* 26 (January-March, 1995), p. 86, and in "*Una Forma per la Parola*", conference held in Recanati (AN) at the Centro Nazionale Studi Leopardiani, on October 5, 1996, and in Lisbon, at Casa Pessoa, on November 16, 1998.

[3] *L'Italia Scacchistica*, published since 1911, is one of the oldest and most authoritative chess magazines in existence today.

[4] See GLI SCACCHI DI LUCA PACIOLI – EVOLUZIONE RINASCIMENTALE DI UN GIOCO MATEMATICO, Edizioni Aboca, Museum di Borgo S. Sepolcro (AR) .

[5] See A. Sanvito in *L'Italia Scacchistica* 1146 (2001).

[6] Pacioli's date of birth is uncertain.

Regardless of whether the invitation came at Leonardo's behest, the encounter between the two men certainly led to mutual esteem, as well as close association and a collaboration that lasted until early 1500, when both spent time as guests at the court of Mantua after abandoning the Duchy of Milan, which had been occupied by French soldiers.

For both, these were years of tremendous – in the case of Leonardo, phenomenal – activity.

By 1499 the artist had finished the *Last Supper* (begun in 1495), painted a portrait of Lucrezia Crivelli, the Duke's new mistress,[7] and completed the renowned decorative programs of the *Sala delle Asse* and several other smaller rooms in the *Castello Sforzesco.* His activity was not limited to painting, however. He continued his study of flight, testing it in an experiment, and delved deeper into the study of military and civil engineering. He also traveled. Although the many trips Leonardo took during his seventeen-year employment by the Sforza are difficult to date, he definitely followed the Duke to Genoa, where in April of 1498, he witnessed a violent tempest, which destroyed parts of the dock before his eyes – as if presaging the storm that would soon strike the Duchy.

Of great importance in these years was Leonardo's study of urban and civic architecture – linked to the renovation of the Dal Verme palazzo, which the Duke had granted to Cecilia Gallerani[8] – but above all, of sacred structures.

These crop up particularly in the splendid drawings that stretch the concept of the centrally planned church in every possible direction.

It is in these works, above all, that we can clearly see an exchange of ideas and information derived from Leonard's friendship and association with Bramante, who at the time was working diligently as an architect at Santa Maria delle Grazie, as well as the influence of Leon Battista Alberti, which reached Leonardo via Luca Pacioli.

Alberti's DE RE AEDIFICATORIA, written ca. 1450,[9] and presented to Pope Nicholas V in 1452, was known of course to artists such as Leonardo and Bramante. Luca, however, had known the author personally and been his pupil in the years when Alberti had served as secretary to the papal chancellery in Rome.

It was in during their sojourn in Milan, in fact, that Leonardo and Luca wrote DE DIVINA PROPORTIONE [ON DIVINE PROPORTION]. The first manuscript copy of the work, preserved in the Biblioteca Universitaria e Pubblica di Genova, was offered and dedicated to Duke Ludovico; the second, from 1498, to Galeazzo di Sanseverino. In 1509, the work was published by the Venetian press of Paganino de Pagani.

DE DIVINA PROPORTIONE contains over sixty illustrations by Leonardo's *"ineffabile sinistra mano"* [incomparable left hand]. Thus spoke Pacioli, who, in the preface, paid homage to *"il più degno tra i pittori, gli studiosi di prospettiva, gli architetti e i musicisti, uomo dotato di tutte le virtù, il fiorentino Leonardo da Vinci"* [the worthiest among painters, scholars of perspective, architects, and musicians, a man endowed with every virtue, Leonardo da Vinci of Florence]."

The admiration was mutual; though Leonardo was less explicit in his praise – in keeping with his character – his manuscripts contain multiple references to Pacioli's works (cvf. Madrid II). He was no doubt inspired by Pacioli when he wrote: *"Non mi legga chi non è matematico, nelli mia principi"* [Let no man who is not a mathematician read the elements of my work] (cvf. Notebook IV, 14r), and certainly drew artistic inspiration as well as intellectual stimulation and ideas for his scientific studies from his association with Luca.

In DE DIVINA PROPORTIONE, the central themes of Renaissance geometry are discussed in terms of their neo-platonic and richly symbolic aspects as well as their practical and functional ones. (See S. Bramly, LEONARDO DA VINCI, p. 510.) In Leonardo's Ms. M 80r, for example, we find a rough draft of five platonic solids accompanied by a related triplet, which he copied from Pacioli:

"E il dolce frutto, vago e si diletto
Costrinse già i filosofi a cercare
Causa di noi per pascer l'intelletto"

[7] *La Belle Ferronière*, Musée du Louvre.
[8] C. Pedretti, LEONARDO ARCHITETTO, Milan: Mondadori Electa, 2007, p. 81.
[9] DE RE AEDIFICATORIA was first published in 1485 in Florence.

[And it's a sweet fruit, charming, and so delightful
Has already compelled philosophers to seek
Our reason to nurture the intellect]

What fascinated Leonardo most was the subject of proportions – both of man and all creation. As he noted: *"la proporzione nelli numeri e nelle misure fu trovata, ma etiam nelli suoni, pesi, tempi e siti e in qualunque Potenza si sia"* [Proportion was found in numbers and measures, but even in sounds, weights, time, and places, and to every possible degree] (K 49r).

The proportional, numerical and harmonic relationship between man and nature is a recurring idea among all the great intellects of the Renaissance, who saw in it the uniqueness of the creative force. Referring specifically to architecture, Rudolf Wittkower magisterially defined this sentiment and manner of confronting the world in his ARCHITECTURAL PRINCIPLES IN THE AGE OF HUMANISM (New York, Norton and Co., 1971, p. 29):

> *"It were the artists, headed by Alberti and Leonardo, who had a notable share in consolidating and popularizing the mathematical interpretation of all matter. They found and elaborated correlations between the visible and intelligible world which were as foreign to the mystic theology as to the Aristotelian scholasticism of the Middle Ages... The belief in the correspondence of microcosm and macrocosm, in the harmonic structure of the universe, in the comprehension of God through the mathematical symbols of centre, circle and sphere – all these closely related ideas which had their roots in antiquity and belonged to the undisputed tenets of mediaeval philosophy and theology, acquired new life in the Renaissance... For the men of the Renaissance this architecture with its strict geometry, the equipoise of its harmonic order, its formal serenity and, above all, with the sphere of the dome, echoed and at the same time revealed the perfection, omnipotence, truth and goodness of God."*

Wittkower thus distinguishes the artists of the humanist Renaissance – above all Alberti and Leonardo - as the originators and promoters of a new mathematical interpretation of the entire sensible world. Wittkower makes it clear that through the study of classical antiquity, Alberti and Leonardo discovered a profound correspondence between the sensible world and the harmonic rules of geometry, which they regarded as confirmation of man's centrality in the Divine Plan.

Yet if this is true, and, as Wittkower writes, Alberti and Leonardo demonstrated this notion in plans and the construction of splendid architecture – particularly centrally planned sacred structures – it is equally true that only with Leonardo – after his encounter with Luca Pacioli – do we have the view that geometry and mathematics go beyond architecture, are intrinsic to everything and must therefore guide all of man's works and (as Leonardo would add) inventions.

Aware of the numerical ratios that govern the harmony of music, Leonardo went further, applying modern mathematical analyses to science and building techniques, mechanics, and later, optics, astronomy, and hydraulics. Nevertheless geometry remained the principal tool of his analysis, while painting his beloved and best understood area of study. In this domain the dialogue between Luca and Leonardo would likewise have been creative since Luca was the custodian of the great teachings of Piero Della Francesca. To this their great forerunner, it was not only perspective but also geometry in the broader sense as well as the geometry of polyhedrons and the ratios binding them that constituted the logical structure underlying every good pictorial composition.

Leonardo assimilated and appropriated Piero's thought, and drew from it new and original possibilities.

He was able to add dynamics to Piero's compositions, which were organized as a linked series of regular polyhedrons rendered in perspective, as if immersed inside each other by some inherent static charge.

Action caught at the instant of its occurrence, pure movement, in a way that only Leonardo knew how to capture.

This was yet a greater difficulty, a challenge, but the result was new and splendid.

These were tough problems, ones that his mind needed, the *"cimento"* [risk]

that Leonardo sought, and in which *"si conosce l'oro fino"* [fine gold is recognized].

Thus in a remarkably short, I would say, feverish time (if compared to Leonardo's usual pace), the artist completed the Last Supper, his supreme masterpiece. In it, he combined his own mastery with the rigor of geometry – the rules of which he had investigated with Luca – to create a new, fascinating composition in which the geometry intended and implicit in the works of Piero della Francesca, was raised to a higher, unsurpassable level.

Later – product of the same creative tension – came the complex geometry of the decorative program of the *Sala delle Assi*, of which it is difficult to find a satisfying interpretation, due to its repeated and heavy restoration.

Finally, the other supreme masterpiece, the Vitruvian Man,[10]– the sole, unique, and even today fascinating humanist interpretation of Vitruvius' passage on the inscription of man in a circle and square[11] (Pl. 1).

Here, in a marvelous synthesis, Leonardo united the beauty and proportions of the perfect human body – the consequence and summation of his anatomical studies – and inserted it dynamically into a harmoniously bound circle and square.

The Renaissance idea of man as the center and measure of all things found its confirmation and foundations, in fact, in a particular and biased interpretation of Vitruvius' words, from which it drew inspiration for representing a man inscribed in a circle and square.

The objectives of Renaissance artists naturally varied; man, perceived as a microcosm at the center of the Universe, represented by the circle, constituted it formally but also symbolically; through his insertion in a square he became a real and tangible unit of measurement, in accordance with Vitruvius' text and intention, since in both the square and the circle, the Romans read a pure, simple, and practical unit of measurement that was related to man: a rod, a step, an arm, etc.

Among the many visual interpretations of this passage, Leonardo's is certainly the best known.

The drawing, which the artist infused with his knowledge of anatomy, owes its good fortune not only to its elegance and beauty, but also, as Wittkower noted, to its symbolism. It differs from the others in that it alone joins the two figures in a convincing manner: the first with open arms and joined legs inserted in the square, the second, with arms raised and legs splayed, conforming perfectly to the circle.

Typically of Leonardo in its beauty, the drawing renders visible the anatomic perfection of the man inscribed in the square down to the detail of his right leg, which is shown with its foot orthogonal to the figure and naturally weight-bearing, as the left, disengaged leg, parallel to the plane of the body, prepares to swing out, a movement, which together with that of the arms, prepares to lead it aloft, and permits the simultaneous movement of the right leg with the natural impetus that releases it.

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[10] The date of the folio is uncertain. Carlo Pedretti, LEONARDO ARCHITETTO, Milan: Editrice Electa, 1978, p. 159, links the drawing to Leonardo's studies of centralized buildings and thus attributes to it a date circa 1490. I hold that the reference to Vitruvius, cited in the long written explanation next to the drawing, Pacioli's presumed contribution, (See note 13) and the focus of Leonardo's interests at the time offer evidence for dating the drawing several years later, to 1497-1498.

[11] Marcus Vitruvius Pollio (ca. 80/70 - 23 BCE) wrote DE ARCHITECTURA in ten books, which he dedicated to Caesar Augustus. Rediscovered during the Renaissance and translated by Poggio Bracciolini (1414), the work served as the basis of Alberti's DE RE AEDIFICATORIA. In Book III Ch. 1, Vitruvius writes: *"Item corporis centrum medium naturaliter est umbilicus. Namque si homo conlocatus fuerit supinus manibus et pedibus pansis circinique conlocatum centrum in umbilico eius, circumagendo rotundationem utrarumque manuum et pedum digiti linea tangentur. Non minus quemadmodum schema rotundationis in corpore efficitur, item quadrata designatio in eo invenietur. Nam si a pedibus imis ad summum caput mensum erit eaque mensura relata fuerit ad manus pansas, invenietur eadem latitudo uti altitudo, quemadmodum areae, quae ad normam sunt quadratae";*
[Now the navel is naturally the exact center of his body. For if a man lies on his back with his hands and feet outspread, and the centre of a circle is placed on his navel, his fingers and toes will be touched by the circumference. Also a square will be found described within the figure, in the same way as a round figure is produced. For if we measure from the sole of the foot to the top of the head, and apply the measure to the outstretched hands, the breadth will be found equal to the height, just like sites which are squared by rule]. DE ARCHITECTURA, trans. Frank Granger, Cambridge MA: Harvard University Press, 1931, p. 161.

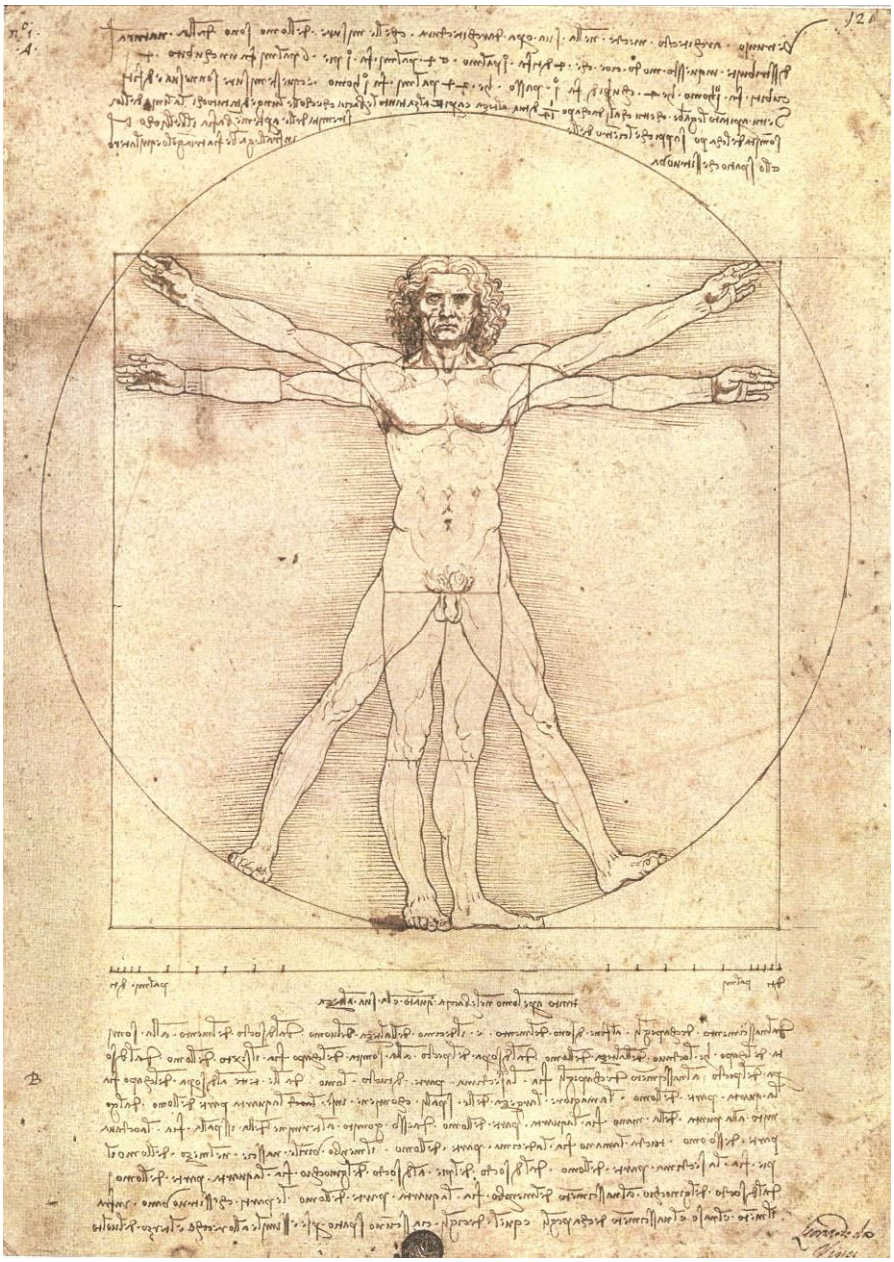

PLATE 1. Drawing known as the *Vitruvian Man*, but also as the *Man of Venice*, due to its location in the Accademia in Venice. Pen and ink over metalpoint, 245 x 344 mm.
The text testifies to how the geometric structure implicit in the *Man of Venice* drawing, which is based on an intriguing, complex, and extraordinary conception of the Golden Section, is the result of deep reflection and elaboration of the principles outlined in DE DIVINA PROPORTIONE.
At the same time, the drawing of the *Vitruvian Man* is a paradigm for the geometric structure and proportions of the shapes of the pieces used in the game of chess.
For these reasons, and also in view of the dubious and often fallacious geometric analyses disseminated by the vast literature on this unique and fascinating drawing, I append to the conclusion of this reading of the Pacioli Manuscript, a study that I hope may help pinpoint the exoteric and esoteric geometrical principles that infuse it.

But it is not only Leonardo who is evident here. Luca Pacioli's input becomes clearly legible when one recognizes the teachings of Piero in the implicit underlying geometric structure within the correlation of the circle and square, in the equilateral triangle (stated in Leonardo's marginal notes to the drawing),[12] and in the merely suggested (one might say subliminally) pentagon in the drawing, in a manner befitting the treatment of a secret *"quintessence"*.[13]

[12] In it, Leonardo explains how a man, standing erect, inscribed in a square, can be transformed into *homo ad circulum*: "If you open your legs so much as to decrease your height by 1/14, and spread and raise your arms 'til your middle fingers touch the level of the top of your head, you must know that the centre of the outspread limbs lie in the navel, and that the space between the legs will be an equilateral triangle." THE NOTEBOOKS OF LEONARDO DA VINCI, ed. J.-P. Richter, vol. I, New York: Dover Publications, 1970, no. 343, p. 182

[13] In fact it can be demonstrated that the side of the square is determined exactly by the height of the regular pentagon constructed at the base of the equilateral triangle described by Leonardo. Leonardo

The *Vitruvian Man* thus marks the historical moment of the encounter between Luca Pacioli and Leonardo and the extraordinary fruition of their friendship and association.

After a period of such intense creative tension, Leonardo witnessed with detachment the bloodless entry of the French into Milan on September 14th.

Whereas the Duke fled and his condottieri and friends either ran away or deserted to the French side, Leonardo remained in the city. To this period dates one of his memos, full of bitterness: "*il duca perse lo stato ella roba e libertà e nessuna sua opera si finì per lui*" [the duke lost the state, its possessions, and freedom, and not one of his works was finished for him].

Leonardo remained in Milan for another three months. In his presence, Louis XII saw the model for casting the stately equestrian monument of Francesco Sforza, visited the *Last Supper*, and expressed his full admiration. (Paolo Giovio records that he inquired whether it would be possible to detach the painting from the wall and take it to France.)

It is unclear whether Leonardo tried to place himself at the service of the king, but after remitting a substantial portion of his savings to the Monte di Pietà in Florence, he departed on December 14th with a small retinue and his faithful companion, Pacioli, for the court of the Gonzaga in Mantua.

Mantua, like Venice several months later, was only a stopover. At the court, Leonardo was beset by requests from Isabella d'Este, who sought a work by his hand in oil, and continued to pester him after he left for Florence.

It was here that Leonardo once again became intrigued by the study of engineering. In fact, on June 21st 1502, he accepted a position in the service of the sovereign of Urbino, Cesare Borgia, *"Il Valentino"*. In early August, Leonardo joined the Duke in Cesena. As his general architect and engineer, he was obliged to inspect all the castles and fortifications of Borgia's newly acquired properties. By March of 1503 he had abandoned Valentino for Florence, where he became involved in the project to extend the Arno's navigability, and was commissioned to paint the lost *Battle of Anghiari* in the Salone dei Cinquecento in the Palazzo Vecchio.

Afterwards, a few additional paintings still engaged him emotionally:

St. Anne, *Mona Lisa*, the second version of the *Virgin of the Rocks*, *Leda* (likewise lost), and finally *St. John*, as well as its controversial variant, *Bacchus*.

To these works we may now add – with the evidence below – the conception, formal definition, and intended production of a splendid set of chess pieces.

These were an extraordinary invention, integrated in a most sophisticated theoretical and analytical framework, allowing us to observe a true and proper evolution of new problems for the game that coincided with the transition of the game's rules from the Middle Ages to the Early Modern era.

An attentive study of the Pacioli Manuscript, allows us to observe this evolution smoothly unfold. Its pages reflect the contrast between Leonardo's inventions and Pacioli's parallel strenuous efforts to collect and illustrate the ancient problems of the game.

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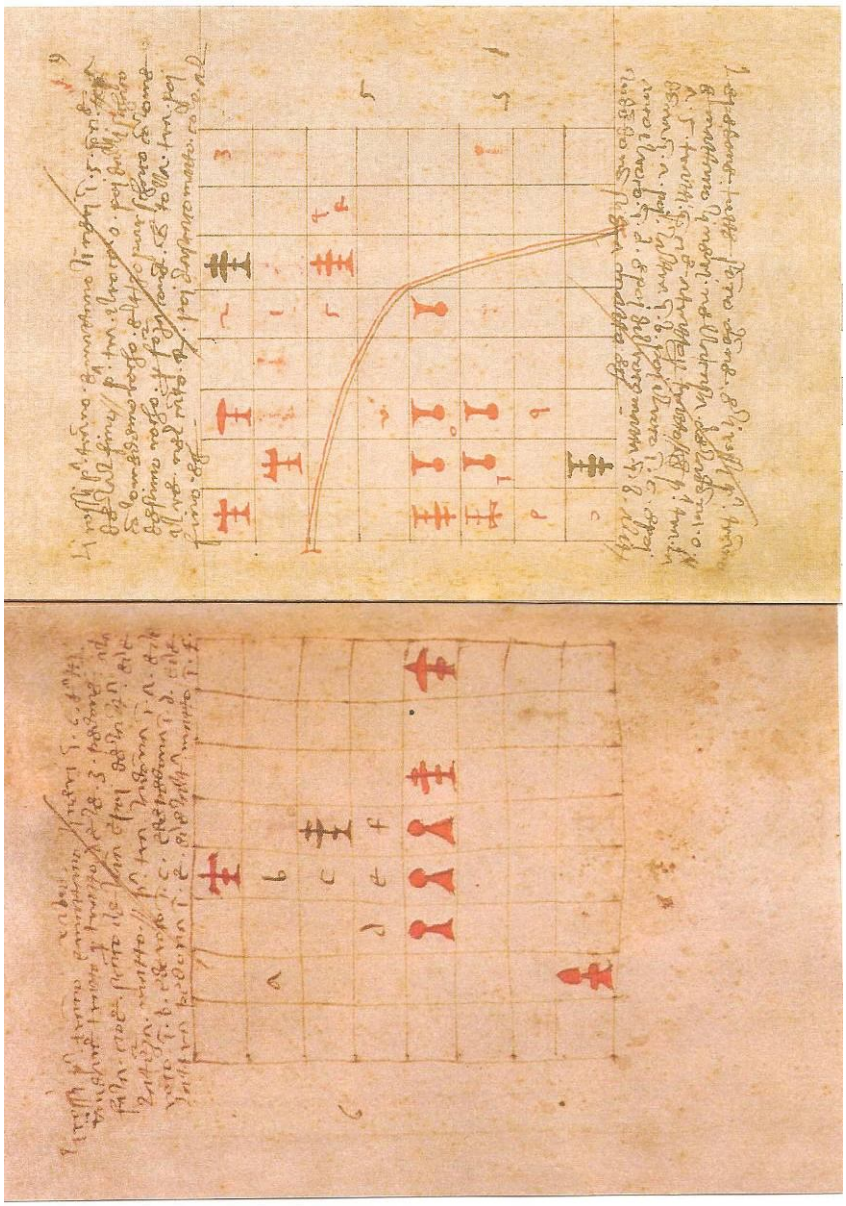
does not explain this point because the construction of the regular pentagon passes through the construction of the golden section. (See the thorough study of the geometric structure implied in the drawing of the *Vitruvian Man*, appended to my conclusion.) This construction demonstrates an absolute familiarity with the use of the Golden Section, which Leonardo acquired only after his encounter with Luca Pacioli and the writing of the DE DIVINA PROPORTIONE; it thus follows that the drafting occurred several years after the date proposed by Carlo Pedretti (see note 10).

FORMAL ANALYSIS

The manuscript was drawn up under the singular conditions described above. To these conditions each specific aspect examined below must be related – particularly with regard to the representations of the pieces, which, delicately rendered with a few brush strokes, illustrate the positions of the game pieces in the problems and the situations of the described matches (Pl. 2).

PLATE 2.



Photograph of the manuscript open to Folios 8v-9r.

On left side, Folio 8v., last page of the first binding, problem entitled *"rabio"*; on right side, Fol. 9r., first page of the second binding, first of the problems to ply with the old rules. The comparison of these sequent pages evidences very well the differences between chess board drawn free hand and pieces "drawn and painted" on the left and chess board drawn with the help of a ruler and pieces "exclusively painted" on the right side, see Pl. 9, p. 18, and followings.

This was also a singular moment in the manner of playing chess. It was precisely between the years 1475 and 1512 that the medieval rules of the game – ambiguous and often diverging from one European court to another – were transformed, codified, and universally recognized and accepted in the definitive forms still used today. It was also in these years, in fact, that the *"alla rabiosa"* or simply *"rabio"* mode of playing was invented with rules now known as "modern" to distinguish them from medieval ones.

It is probably not incorrect to see evidence of the Renaissance in the confrontation between the modern rules and the confusing and somewhat contradictory quality of the earlier ones, for this was a period in which all human

activity was subject to critical analysis, greater self-consciousness and confidence in the individual's ability to clarify, comprehend, and control all aspects of his life, even in a game, which, after all, was meant to be not only that but also a challenge, the meeting of logic, memory, and intuition, that is, the meeting of intelligence and virtuosity. In a world exalting beauty and valor, contests and confrontations, challenge and the opportunity to flaunt intelligence were characteristics that granted the game the success and interest that any courtier drew from playing it well.

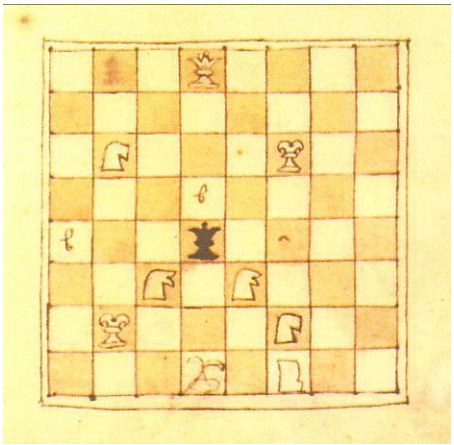
In view of the slow pace that the former rules imposed on chess and the quarrels easily roused by their variability, it is not surprising that the game evolved in a positive direction after the *"alla rabiosa"* mode was introduced. Since that time, in fact, the rules have undergone no further changes. Certainly complicit in this shift was the widespread diffusion of the aristocratic game to all the courts of Europe. Documented in countless images, the game's popularity is also testified by the extensive collections of chess problems that have survived, and in which we can witness the gradual affirmation of the new manner of playing.

From this precise period, twelve texts discussing the problems of the game -- including the recently discovered one by Luca Pacioli -- have survived. These range from the Valencian manuscript SCACHS D'AMOR of 1475, which expounds on problems played with various medieval and several *"alla rabiosa"* rules, to the Hispano-Italian DAMIANO of 1512, which covers only problems played *"alla rabiosa."*

In terms of the forms of the pieces, the description of each problem in such texts (usually presented more or less schematically with the formula "in this set and explicit [number of] move, white checkmates black") is accompanied by a drawing of a chessboard on which is indicated the starting position of the game pieces. As a rule, the pieces on it are indicated either by name or an ideogram, similar to those still used today to illustrate game situations in specialized magazines or books.

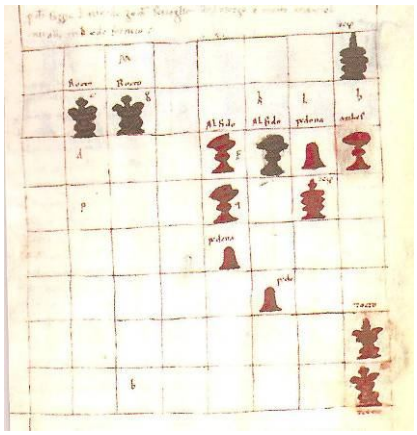
Only three of the twelve known texts deviate from this model. Among them is the Pacioli manuscript, in which the pieces are shown not as abstract ideograms but as actual pieces. In the other two, well known to those who study the game and its history – the Ms. Cod. Cartaceo 2851, preserved in the Biblioteca Riccardiana in Florence, and Ms. Cod. Membranaceo 128b, in the Biblioteca Reale of Turin, which is dedicated to Borso D'Este (Pl. 3) – the pieces resemble those in paintings or book illustrations depicting the game. The idiosyncrasies of their forms are and always have been regarded as nothing more than aesthetic embellishments, especially as the texts were prepared as works of special prestige for personages known not only for their passion for the game but also for their wealth and elegant refinement.

PLATE 3.



Ms. Codex Cartaceo 2871, Biblioteca Riccardiana, Florence.

Islamic-style chess pieces.



Ms. Membranaceo 128 Biblioteca Reale, Turin
Dedicated to Borso D'Este.
These figures are similar to those painted by Lucas van Leyden; see Plate 8.

In the third manuscript, which may now be added to the other two, however, the illustrated pieces have novel, unprecedented forms – seemingly born out of thin air. Although this manuscript would certainly have been dedicated to a person of great culture and exceptional intelligence, such as Isabella d'Este, the

Marquise of Mantua, one of the wealthiest and most sophisticated courts in Europe, it is clear that the author or authors (as perhaps it is better to anticipate the plural) intended it merely as a preparatory text.

The manuscript, in fact, lacks a title, a dedication, and any notable graphic organization, save a simple schematic order with a block of text at the top of the page and a schematic chessboard at its center. In addition, the text is written in *"mercantesca veloce,"* a script commonly used for commercial operations and certainly not for a text intended to be presented at court.

It contains another peculiarity, however. All twenty-four sheets comprising the booklet are approximately 16 x 22 cm. and folded in half. Each therefore amounts to two folios of 16 x 11 cm, that is, four smaller pages, which together add up to 4 x 24, a total of 96 pages. Each one contains writing and a chessboard, but as two distinct "matches" are depicted on 18 of the chessboards, the total number of problems presented is 114, that is, 96+18. The descriptive text in these cases appears above the chessboard and sometimes upside down in such a manner that the booklet needs to be turned 180° for the text to be read. In other cases, the second match is explained at the side of the drawn chessboard, in which case the manuscript needs to be turned only 90º.

In sum, everything indicates that we are dealing here with a rough draft, an initial text that was to be followed by a new and definitive version with a title, appropriate graphic form, and a dedication, which had already been sketched out and articulated by Fra Luca Pacioli in his preface to DE VIRIBUS QUANTITATIS.

All the same, if this was simply a text or rather a collection of texts meant for personal use, why did the author go through the trouble of applying color to the pieces instead of simply charting them more rapidly with letters or symbols?

The answer to this question is related to the answer to a more fundamental question; having confirmed that the pieces' forms are entirely original, who in fact conceived them?

In order to answer this question, it is necessary to recall that back then, as now, chess pieces were divided into two basic and highly distinct categories: "one consisting of figurative, decorative, and representational pieces; the other of simple pieces used in habitual play, which lack naturalistic forms, and whose features functioned simply to differentiate them from each other."[14]

This subdivision has early origins, and – at least in the case of the commonly used pieces – probably dates back to the symbolic schematization imposed by Arabs on the highly figurative original pieces of the Indian game due to the Islamic ban on representational art.

Noteworthy too is that in the late fifteenth century, the forms of the pieces commonly used in the game were not codified as they are today, but were mutable and sometimes even left to random interpretation.

This situation led to complications that were aggravated by the new early Renaissance taste for simplified forms. Moreover, because the pieces were now more frequently manufactured on the lathe, the sculpted elements, which had naturally granted more easily distinguishable shapes to the pieces, were further simplified.

Of particular relevance is the problematic distinction between the bishop and the knight,[15] which lasted until the late nineteenth century when the Regènce set became the norm. The Regènce, in turn, was followed by the set known as Staunton, (officially born on September 29th 1849), still used in all official matches today.

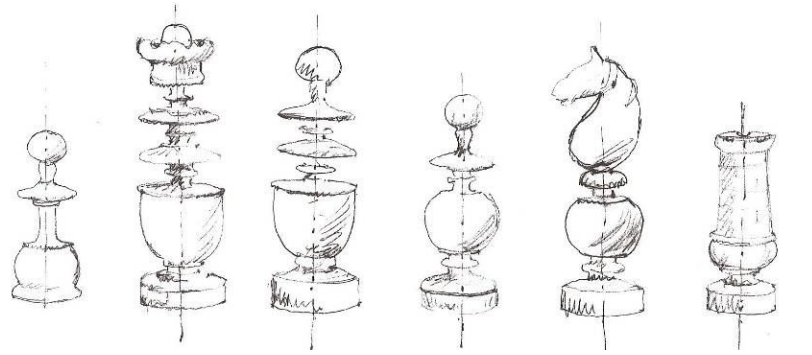
In a Regénce chess set, five pieces – the King, Queen, Pawn, Rook, and Bishop – are made on a lathe, while the Knight has a carved horse's neck and head, which are attached to a wood-turned base. In Staunton chess, the Knight too has a carved neck and a horse's head attached to a wood-turned base. Yet in order that the pieces be instantly identifiable, only the pawn is made exclusively on the lathe; the other four are manufactured with labor seemingly simple but impossible on a lathe, especially in the case of the cross crowning the King, the reliefs on the

[14] A. Sanvito, *Figure di Scacchi,* Milan: Mursia, 1992.

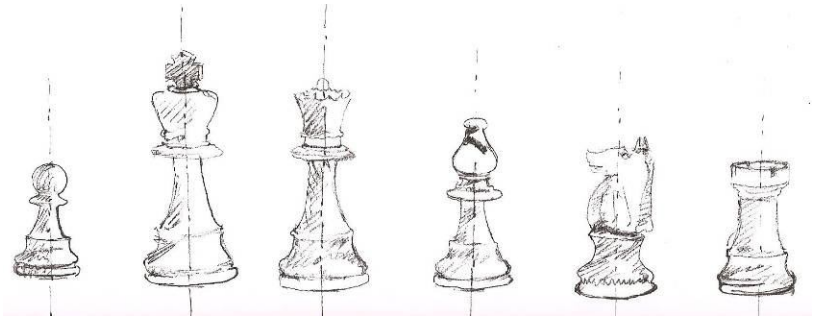
[15] On this issue, see Roberto Cassano, "14 antichi pezzi del Museo Civico di Albano," in *L'Italia Scacchistica* 1193 (2007).

Queen's circular crown, the oblique slash on the Bishop, and the hint of crenellation on the Rook (Pl. 4).

PLATE 4.



Pawn King Queen Bishop Knight Rook
RÉGENCE CHESS SET



STAUNTON CHESS SET

The chess pieces depicted in the manuscript undoubtedly belong to the more commonly used category. By the will and aesthetic choice of their creator, however, or merely for the sake of a simpler manner of representation imposed by the small scale of the folio's format, the pieces are particularly simple and stylized. All six different figures, in fact, could be manufactured on a single lathe; this is certainly the result of a precise formal choice and intentional design. (Pl. 5)

PLATE 5.

Pawn King Queen Bishop Knight Rook

Queen

The Queen is the only one of the six game pieces drawn in two different manners.
The one in the upper row, the "*fountain base* type", is represented only among the pieces that are "exclusively painted;" the one below is of the "*plume*" type, represented exclusively among the pieces that are "drawn and painted." See pages 23 and 36.

The figures of the King, Queen, and Pawn are the easiest to identify thanks to the uniqueness and compulsory presence of the first, the unusual design of the second, and the frequent representation of the third. The other three figures - the Rook, Bishop, and Knight - are more difficult to identify in the manuscript because they bear no resemblance to other known, conventionally used game pieces.[16] Also noteworthy, is that though the six figures are always recognizable

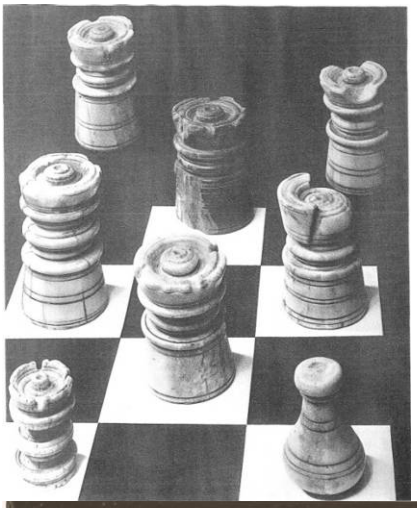
[16] A brief conversation with Professor A. Sanvito, who was both forthcoming and courteous, allowed me to identify them with certainty, on the basis of a correct and not always easy interpretation of the illustrated game situation.

and distinguishable from one another, all are represented with slightly inconsistencies throughout the manuscript. This seems to be the case not only, or at least not exclusively, because the figures were drawn by hand, but also because they were executed with a certain disregard for the game pieces that they were meant to represent, probably because, as noted, this was a draft for a treatise that would be redesigned at the moment it was decided to send it to print, and also, as we shall see below, because it was a work executed by two hands.

Whatever the case, the median can only be determined by comparing representations of the same game piece. This is a task similar to extrapolating the specific characteristics of a letter from the various ways in which it has been written over and beyond the idiosyncrasies of a particular script.

The figures on some pages of the manuscript are drawn with greater accuracy and detail. These stand out from the other, less defined ones, which are either rendered carelessly or with obvious flaws. I compared the six game pieces of the manuscript, based on their profile and median (Pl. 5), to analogous ones illustrated in several published works on the historical evolution of the pieces' forms, particularly A. Sanvito's FIGURE DI SCACCHI (op. cit.) and Hans and Siegfried Wichmann's SCHACH, Munich: Verlag Georg D. W. Calleey, 1960. In these, images of turned chess pieces either from the Renaissance or earlier are rare and often rather crude if compared to the more slender and elegant ones in the manuscript (Pl. 6).

PLATE 6.



Sample representations of turned pieces used in the late fifteenth century.

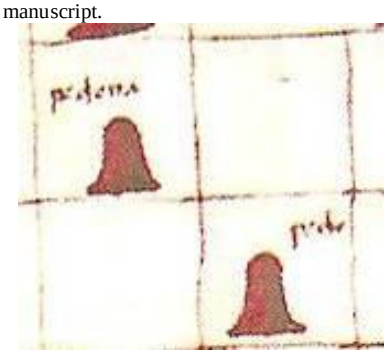
Detail

Lucas van Leyden, *The Chess Game*, 1508

Sometimes the manuscript's Queen seems to bear some resemblance to representations of the same piece common in that period or earlier, but on closer inspection the features that make it similar are merely superficial and suggested by the fact that they pertain to figures manufactured or designed to be manufactured on a lathe.

Only in the case of the figure of the pawn, in use by the late 1400s, do some of the representations come fairly close to those in the manuscript (Pl. 7).

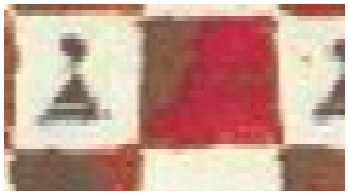
PLATE 7.
Comparison of the forms of pawns used in the late fifteenth century and that depicted in the manuscript.



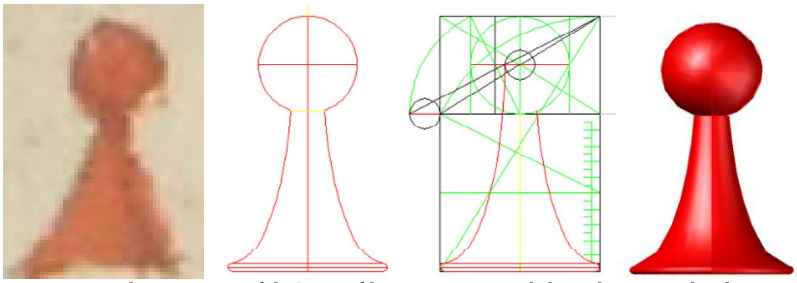
"*Pedona*" – Pawn, from Cod. Memb. 128 in the Bibl. Reale, Turin

Ivory turned pawn, second half of fifteenth century

Pawn, detail
In Lucas van Leyden, *The Chess Game*
Staatliche Museen, Berlin



Pawn from Alfonso X's *Libro des Juegos*
Sec. XIII, Biblioteca dell'Escorial, Madrid



Pawn in Pacioli manuscript, fol. 8v. Profile, proportions and three-dimensional volumetric reconstruction.

Finally, the King, Rook, Bishop, and Knight contain features that are totally new. It must be underscored that already in this preliminary analysis, the six game pieces of the manuscript -- despite having different forms that make them clearly distinguishable from each other -- are, as a group, harmoniously and unequivocally coherent as a carefully defined and planned whole.

We will see in the details of the relative dimensions and relationships of the parts that constitute each piece how the formal unity and harmony of the whole are the result of precise geometric rules and carefully reasoned proportional logic.

In any case, what is confirmed and what was already anticipated in the earlier comparison between the Pacioli manuscript and the other two coeval ones in which game pieces were indicated through representational rather than symbolic means, is that the chess set depicted in the manuscript is utterly novel and original both in its own period and ours. It was conceived organically and expressly to illustrate the game situations, problems, and matches described in the manuscript, and appears only in this manuscript.

We return to the question as to who is to be credited for the formal definition of the pieces, which, as we have seen, totally disregards the form of the pieces of a game already ancient and in many respects codified. And we ask it fully aware of the fact that we are implicitly excluding Luca Pacioli as the inventor since he was a mathematician and theoretical geometer who never applied himself to inventing a new form for anything. So much so that more than one historian – beginning with Vasari – accused him of lacking originality even in the restricted domain of his own expertise, and of being above all a promulgator of ideas not his own.

We cannot help but intuit that Luca, **if working alone**, would at best have used existent and more widely employed forms in order to render his ideas more

clearly, since he was concerned with illustrating the strategy and tactics of the game, with particular emphasis on its mathematical and geometrical aspects.

Did he work alone, however?

We know and must recall that Leonardo was at Luca Pacioli's side, and that the two had collaborated on the draft of DE DIVINA PROPORTIONE in these very years. Perhaps then, these pieces are of Leonardo's design – his invention.

That my answer is affirmative, I have already intimated in my title, though I do not deny that for now the picture that I have sketched and the evidence (indication merely of time and place) I have gathered, offer but an indefinite answer.

With regard to this incertitude, I paraphrase Enrico Magni-Dufflocq, who – in addressing the debate over the invention of the violin in his essay *"La musica vinciana"* in LEONARDO DA VINCI (Ed. I. S. G De Agostini 1956) – observed:

"[I]t is impossible that he did not know about a thing of this kind in a house where he came as a friend; it is impossible that knowing what the lutist was making, Leonardo did not offer a hand, a word, a drawing, or a piece of advice."

So I too may add that it is impossible that in the months that they spent together, Leonardo, aware of the book that Luca Pacioli was writing, took no interest in it, did not intervene, and left to others (and to whom?) to come up with a completely new series of pieces.

A series of chess pieces, with something extra – as was typical of his innovative and inventive mind: a modern quest for a simplification of form that would facilitate the easy and rapid production of the pieces of the aristocratic game by means of a simple lathe without, however, sacrificing aesthetic value. Perhaps it amused him to create a model for this series – in keeping with his habit – which he later used as an exemplar for the pieces depicted in the manuscript. If this was, in fact, the case, then he would have used this set to play with his friend Pacioli. With these uniquely elegant and quintessentially modern pieces arranged on the surface of the chessboard, the two of them could have reconstructed the oldest, best known and cleverest matches described in the treatise and have imagined unprecedented symmetries in the new schemes and problems of the game.

Given that he often considered and wrote about his wish to derive commercial profit from his inventions, Leonardo may also have considered manufacturing pieces.

The pieces illustrated in the manuscript, however, have remained unknown not only because no one had seen anything like them before their rediscovery, but also because their forms, designed to be executed on a lathe, were in actuality very difficult to manufacture on one. The relationship between the slim, central bodies and the elements both encircling and projecting from them in elegant and proportional sequences is too refined and exaggerated for production by mechanical means. This problem too is characteristic of Leonardo, who in so many of his works always preferred to try utterly new means – even risky ones – for executing his inventions than to fall back on ordinary, safe, and tested methods that seemed to him insufficient for achieving the degree of perfection he sought.

Though he certainly never followed up on this plan, Leonardo returned to the subject of chess, particularly to the issue of their manufacture ten years after he left Milan. In late 1510, in the margins of a list of ingredients of a complex *"mistura"* or [concoction] (some type of plastic substance; he sometimes experimented with artificial materials for making pearls or semi-precious materials), Leonardo noted that it could be used for making *"manichi per coltelli, portapenne, scacchieri"* [knife handles, pen holders, chess pieces, etc] (C. A. 861v). Thus according to a scenario based on circumstantial evidence, Leonardo may have designed and produced a set of chess pieces that could have been manufactured on a larger scale.

The possibility of realizing after five hundred years the form that Leonardo conceived lies in a thorough formal analysis of the pieces and a rational examination of the ideas underlying their design and that of the schematized chessboards on which they are represented.

Before taking up this challenge, I would like to clear the field of certain preconceived objections that may surface in response to my hypothesis.

I know that many, aware of the manuscript's discovery and familiar with the artist's biography, have wondered whether Leonardo played any part in its

conception. Some – in truth but a few – have replied with an immediate no,[17] uttered with a degree of certitude corresponding to the lack of justification in their denial. Others – most, I would say– have responded with a hesitant and hopeful "maybe."

Only a few have presumed that Leonardo was so exclusively dedicated to transcendent thoughts and alienated from everyday reality that he would never have devoted himself to something as mundane as the design of game pieces. Such an assumption would never even be entertained by anyone with the slightest knowledge of the artist's seven thousand surviving folios of studies, observations, and drawings, which reveal his simultaneous attention to both complex and simple problems. Indeed in the process of fixing on paper each object of his scientific research or curiosity – an exercise that Leonardo practiced in a rather disorganized fashion throughout his life – he treated as equal works of art and unique inventions, odd reflections on daily life, and solutions to extremely complicated problems, which he described, and more frequently sketched directly besides other at times banal ones.

Thus nothing in the end rules out the possibility that Leonardo devoted himself to the definition of chess-piece forms with the same intelligence and sensitivity with which he dedicated himself to so many other kinds of work.

Conversely, that among the infinite things to which he dedicated his attention were chess pieces is not only possible but highly probable, given the broad diffusion of the game in the courts of Europe and the fact that the aesthetic quality of the pieces used at the time would have fallen far beneath Leonardo's aesthetic standards.

We therefore begin with a stylistic analysis of the pieces, starting with the pawn.

Of a complete chess set of thirty-two pieces, half are pawns. Consequently, in order to save time and reduce costs, the pawn is the piece, which, in the manufacture of common chess pieces, has historically been simplest and best adapted to production on a lathe. Due to its predominance, in a set in which all the pieces, like those in the Pacioli Manuscript, are turned on a lathe, it is the one whose form most determines the overall aesthetic. It is thus logical to presume that in designing a set, any inventor will begin with this piece.

Even though there are very few surviving turned chess pieces, particularly pawns, dating to the time of the manuscript or earlier, there are many depictions of chess problems in treatises on the game, as well as in prints and paintings of various sort dating back to the second half of the fifteenth century when the game was rapidly spreading, in which turned pieces are clearly distinguishable (Pl. 6).

In these, it is easy to distinguish pawns -- easily recognizable because they outnumber all other pieces – produced, or with the potential to be produced, on a lathe. These generally assume the shape of a little bell, occasionally surmounted by a small sphere (Pl. 7). This is a simple and logical shape, in which it is not unusual to see a reference to a standing human figure, and one certainly dictated by simple utilitarian demands: stability on a chessboard, ease of handling, and last but not least, facility and speed of production.

Even today the pawn of a Staunton set has a similar form. But now it is slightly "embellished" and one might even say disfigured; the sphere on top is proportional to a slightly larger base, beneath which is a small disk that crowns the body supporting it and which also makes it resemble a little bell that is not smooth but adorned at its base by two ridges (Pl. 4). Nevertheless with Staunton pieces we are in the nineteenth century, when certain formal redundancies were fashionable.

The Pacioli chess piece designer modified the pawn (as noted, the only one of the six pieces based on a codified image) in a simple and sophisticated manner by altering the relationship between the diameter of the sphere on top and the height of the bell-like body that supports it, as well as that between the latter's

[17] Thus, for example, the Spanish chess scholar, José Antonio Garzón, commenting on the rediscovery of the manuscript, notes: "When I showed the drawing to my friend, the bibliophile and writer Rafael Solaz Albert, a drawing specialist and expert on Leonardo's work, he expressed with certainty that he did not believe this was a work by the Renaissance genius." (This observation, made several days after the identification of the manuscript, J. A. Garzón seems now to have rejected. See his new study on the manuscript.)

height and the diameter of its base. The ratio thus shifted from 0.3:1 and 1:2 to around 0.6:1 and 1:1, respectively. The result, simple and extremely elegant, betrays the judicious use of the Golden Ratio while leaving the older features of the piece recognizable. (Pl. 7)

Before turning to the form of each piece, the manners in which they were represented, their relative dimensions, and the relationships between them in order to assess their aesthetic value as individual pieces and as a whole, let us examine the chessboards on which they were arranged as well as all the other graphic elements that flank them on each page.

In the over one hundred chess maneuvers or *"partiti"*, the illustrated pieces, often finely painted one by one with a few strokes of the brush tip, are shown on a schematic chess board. These are drawn more or less at the center of each page, slightly closer to the top than the bottom and noticeably closer to the inner margin and the sewn binding than to the outer edge. Traced lightly in sepia, they are inserted beneath the text or between two blocks of text, one per page and of two different sizes. (As we shall see when we examine in detail the signatures that compose the manuscript, the two different sizes are a bit under 85 square mm per side for the eighty chessboards (2 x 34 plus 2 x 6) drawn on fols. 1r - 34v and 43r to 48v, and a bit under 60 square mm per side for the sixteen (2 x 8) drawn on fols. 35r - 42v. They are generally executed in the same ink as that used for drafting the texts. The ink's intensity, however, is inconsistent; sometimes it is lighter, sometimes darker. The color too changes; sometimes it inclines towards greyish-yellow, sometimes towards a brilliant red. Discrepancies between the color used for writing the text and drawing the chessboard are likewise visible on various pages.

The boards were drawn in two clearly different manners. On 96 pages, 72 chessboards were drawn with the help of a ruler, while 24 were executed freehand.

Those done freehand are all 85mm per side. The ruler-drawn chessboards, on the other hand, come in two sizes; sixteen are 60 mm per side, and 56 are 85 mm per side.

The ruler-drawn chessboards are constructed in a consistent but not identical geometric manner. Especially in the case of the larger ones the two horizontal lines defining their tops and bottoms run across the entire folio from the left to the right margins. In the smaller ones, on the other hand, both the horizontal and vertical lines defining the chessboards extend beyond them to the margins.

One can see that these lines were cut when the folio was trimmed. The seven vertical and horizontal segments that form the chessboard's grid lie within the square created by the junction of these two or four lines. Whereas in the larger chessboards, the segments were drawn with a ruler, in the smaller ones they were done freehand but with great precision and accuracy – so much so that at first glance, they too seem traced with a ruler.

Finally, on certain pages, it is evident – both on the recto and corresponding verso – that dots aligned with the inner segments served as guides for tracing them (Pl. 8-2) All this makes it clear that the chessboards were drawn on larger pages that were then cut to create the individual sheets, which, folded in half, formed the pages.[18]

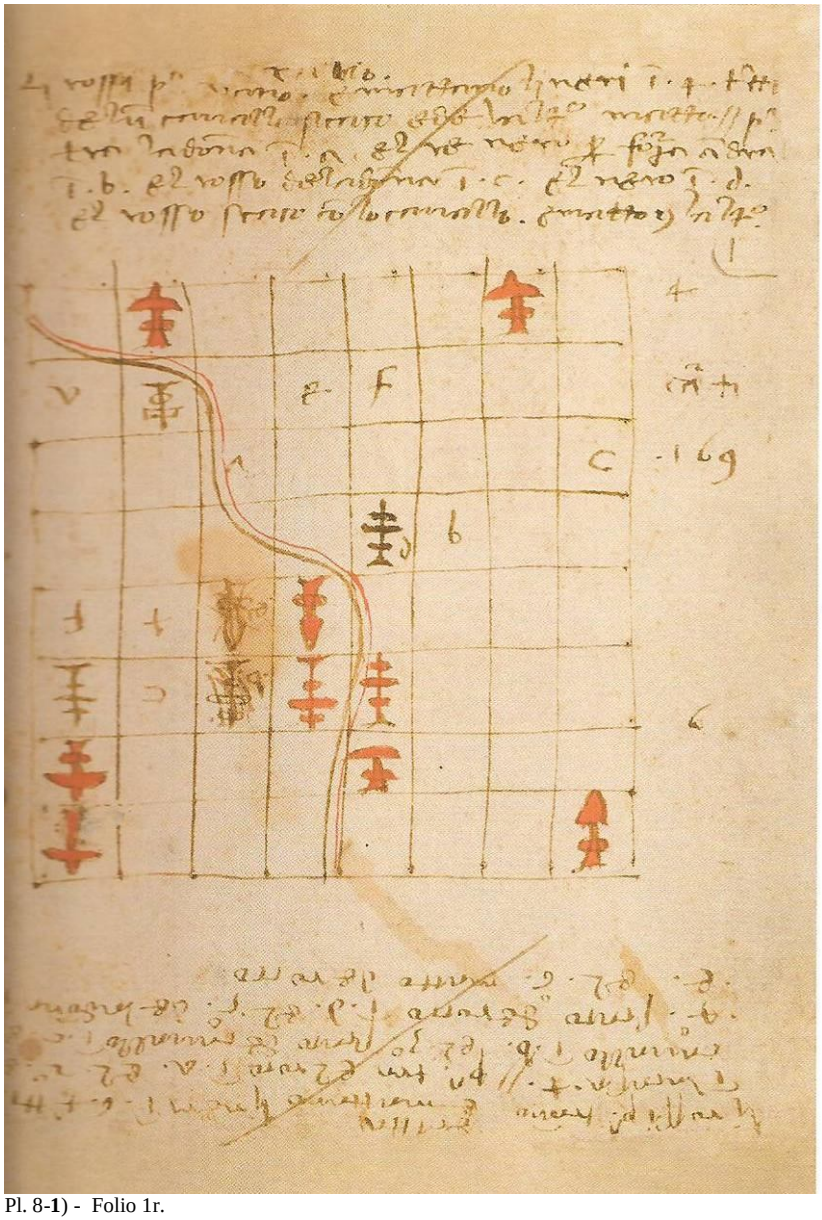
The freehand procedure used to draw the other 24 chessboards is utterly different. These are all the same size as the larger ruler-drawn ones. Each is set on the page in the same position as the others, that is, slightly higher than center and pushed a bit towards the inner margin. They are drawn entirely freehand, without the help of preliminary lines but with four dots marking the top and seven along each of the four sides.

These 32 dots (4+7x4) are made with great precision but certainly by eye and freehand.

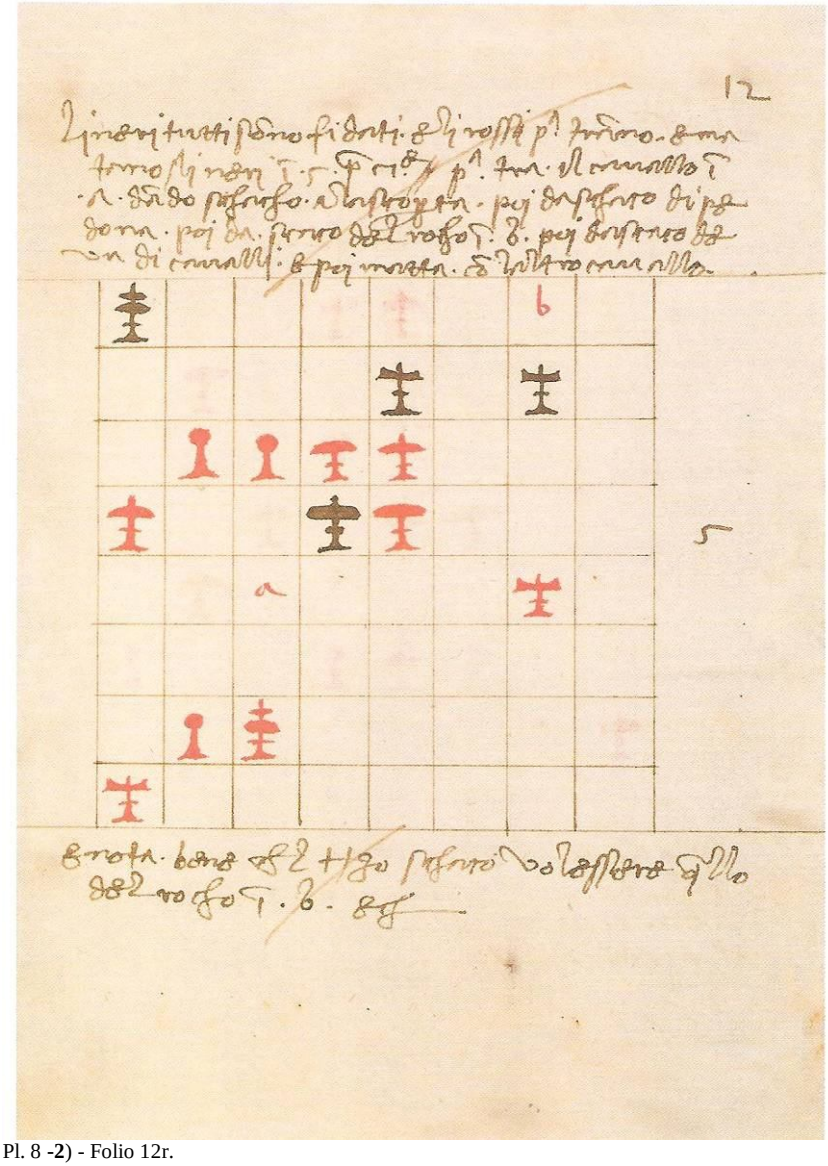
[18] The dots in question could have been made on the paper by pin holes in order to ensure the correspondence of the chessboard drawings on both sides of the same folio. This is not the only method of making certain that the chessboards on the recto and verso of the same folio are in an identical position. Examining the original, one can easily determine the system used and the number and dimensions of these pages. Such a reconstruction is not strictly necessary to the task I have set myself, so I have left it aside. I note solely for those who wish to delve deeper into the problem that the manuscript is composed of 24 pages; as these measure 150 x 220 mm they could have been cut from 6 folios measuring 310 x 440 mm, the precise dimensions of a type of paper known as "rezzuta," produced by paper mills in Fabriano in the fifteenth century.

The actual chessboard pattern is formed through the juncture of nine vertical and nine horizontal lines that join the aforementioned dots on opposite sides. These line segments were all drawn by hand in one swift stroke. The precise point at which the pen touched the paper is always distinguishable because it invariably coincides exactly with one of the dots on the sides. Due to the swiftness of its execution, however, the line rarely reaches the corresponding dot on the opposite side. These segments were drawn so quickly, with so little heed for the outcome, and so often at a parallel slant to each other that they give the impression of having been drawn on a sheet held by hand rather than resting on a flat surface (See Pl. 8-**1** Folio 1r., and Folio 8v., on previous Pl. 2,).

PLATE 8.

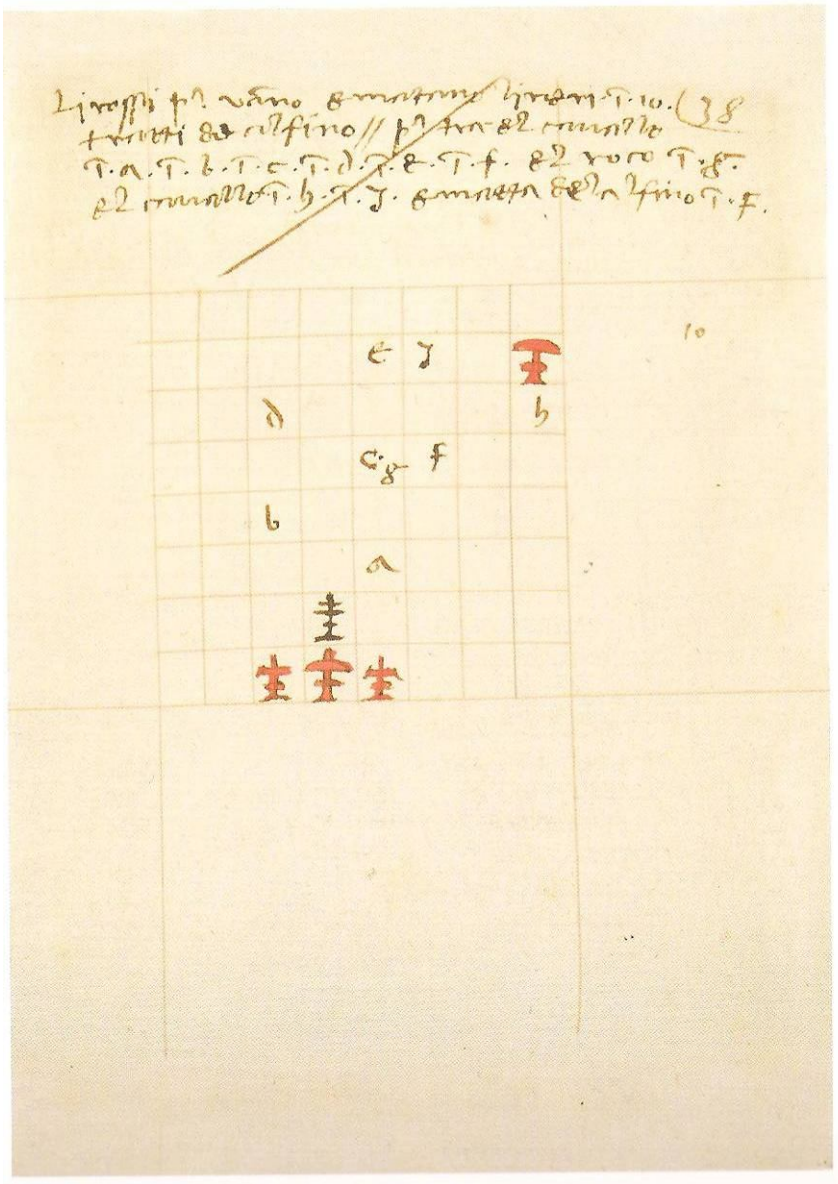


Pl. 8-**1**) - Folio 1r.
Chessboard, 85 mm. per side, drawn entirely freehand.
The nine drawn dots that divide each of the four sides into 8 nearly equal parts are clearly distinguishable; these too were done freehand without the help of a rule. The lines that join them to form the checks of the chess board were sketched very quickly in one stroke from right to left and from the bottom up.
Numeration appears above, to the right, beneath, and after the drafted text.
This manner of representing the chessboards appears on the first 8 successive folios, on folios 33 and 44, parts of the same sheet, first and last folios of the fourth fascicle and on folios 46 and 47, parts of a same sheet, respectively second and third folios of the fifth fascicle. (See the fasciculation).
On these chessboards the pieces are always and without exception “drawn and painted.”



Pl. 8 -**2**) - Folio 12r.
Chessboard, 85 mm per side, drawn entirely with the help of a ruler.
This manner of depicting the chessboard appears on 48 successive pages from folios 9 through 32, on parts of the twelve sheets of the second and third fascicle. The numeration appears in the upper right and was inserted freely and was not separated by parentheses, before the text was drafted. On these chessboards the pieces are always "exclusively painted.", with one only exception, a problem drawn on a portion of the chessboard of folio 25r..
One notes the horizontal lines of construction that outline the chessboard and reach the edges of the folio. To the right, at the side of the numeral 5 that indicates the number of moves needed to solve the proposed problem, can be seen the very lightly sketched dots the served as guides for drawing the internal lines of the chessboard – the same ones that appear in the same position on the verso of the folio. This same manner of depicting the chessboard appears on folios 34 and 43, that are parts of a same sheet and on folios 45 and 48, also part of a same sheet, respectively, of the fourth and fifth fasciculation. On these chessboards pieces are "drawn and painted." (See the fasciculation Pl. 10).

Pl. 8 -**3)** Below - Folio 38r.
Chessboard of 60 mm per side, drawn with the help of a ruler
One may note the outer horizontal and vertical lines of the chessboard that are drawn with a ruler while the internal lines ones are drawn freehand.
The numeration appears on the upper right and was inserted before or at the same time as the text was drafted, and is separated from the text with a parenthesis.
This manner of depicting the chessboard appears on 16 successive pages, from folio 35 through folio 42 at the center of the third 6-sheet fascicle. See the fasciculation.
On this chessboard, the pieces are always and without exception "drawn and painted."



It is on these boards that the chessmen were drawn in the manner in which we examine in detail below.

Noteworthy too is that aside from the illustrated chessboard and the text describing the problems related to the match depicted on it, three other bits of information are jotted down at the side of each drawing.

First, there is nearly always a cipher indicating the number of moves needed to resolve the problem presented. This number is always reiterated in the text that describes the match, and is missing in only those three folios where the solutions to the problems represented do not depend on a specific number of moves.[19]

Often there are short notes that refer to numbered pages, such as *"carti 169"* (fol. 1r) –Pl.8 -1), *"aliam habes carti 16"* (fol. 16r), *"similis carti 37"* (fol. 28v), and *"carti 180"* (8r). Finally, on fols. 41r and 41v we find *"idem habes in meis quinternis carti 103"* and *"idem habes in nostris,"* respectively words that make it crystal clear not only that many problems discussed in the manuscript had been previously organized in files, but also that Luca Pacioli had not acted alone since some of these notes are not his own *"meis"* but ours *"nostris"*.

Finally, indicated on the recto of each folio are page numbers that progress without interruption from 1 to 48. This pagination is noted in three different manners: with a number written in the upper right hand corner of the page above the text; with a number written in the upper right hand corner but inserted into the

[19] These are one of the two matches on fol. 17r (the "giro di cavallo" problem), one of the two matches on fol. 17v (the problem of opposition) and match 82 on fol. 34v (the ancient problem only of queens).

text and aligned with its first row, though separated from it by a bracket; and finally, in several cases, beneath the text and flanking the upper right side of the chessboard. In the last cases too it is framed on the left and below by a short bracket. These three modes of pagination appear in distinct parts of the manuscript, and, as will become evident, are consistent with all other features examined here.

The first mode – the number written above the text high on the upper right hand corner of the sheet – appears in all the central folios, from 9 to 32, and reappears on fol. 45. The second mode – the number written on the upper right hand corner but inserted in the text and aligned with its first rows – appears from fol. 34 to fol. 43, and again on fol. 48. The third mode – the number written beneath the text and flanking the upper right side of the chessboard – appears on the initial sheets, from fols. 1 to 8, as well as on fols. 33, 44, 46, and 47.

Furthermore, the descriptions of the one or two matches depicted on the chessboards are placed, as noted, above or below, and also to the side of the drawings in vertical rows, occupying the spaces left free by the chessboards and occasionally overlapping their borders.

One peculiarity that proves what we have already asserted when noting that the descriptions reiterate the number of moves indicated at the side of the chessboards is that the formulation of the texts followed on the drawings of the chessboards and illustrated matches.

Finally, nearly all the texts – 101 out of 114 – contain deletions made by decisive pen strokes slanting towards the right, and drawn from bottom left to top right. In the thirteen matches where this sort of stroke does not appear, a small x inclined to the left[20] appears on the border of the sheet beside the chessboard.

Summarizing the information presented here, one may affirm:

1) That the compilation of the manuscript follows a precise order:

- In the first stage, 24 sheets were folded in half to create 48 folios. Eighteen of these sheets were recut from larger sheets on which chessboards had been drawn on recto and verso. In the other six cases, the chessboards were drawn after the sheets had been cut and folded.
- In the second stage, the pieces were drawn on the boards to illustrate the starting positions of chess problems, some of which were already known and noted in other documents, while others were novel and designed to be played with the new rules. At the same time, the number of moves needed to resolve the problem was noted at the side of each match.
- Next the sheets, which were folded to form two folios and fasciculated, were arranged in a precise order, numbered, and filled with text.
- Finally these were edited and corrected in the two manners described above in order to certify that they had been checked

2) That the chessboards, pieces, and text were executed at separate, consecutive moments in this sequence (the drawing of the chessboards, the drawing of the chess pieces, the drafting of the text) is important too and fits the claim we make to answer the question we have posed, namely, whether the chess pieces were Leonardo's invention.

We now turn to examining the manner in which the pieces were drawn.

Opposing teams, today distinguished by black and white, are differentiated here too by contrasting colors: a dark sepia, nearly grey, and a deep, brilliant and well preserved crimson.

Each game piece is painted within the square of the chessboard that it occupies according to the rules, strategy, and tactics of the given game problem, and is slightly detached from the base line of the square in such a way as to be clearly legible.

The King, the Bishop, the Knight, the Rook, and the Pawn (or *"Pedona"* [she-pawn], as Luca Pacioli peculiarly calls the figure) have various distinct forms.

As a group, however, they are harmonious and consistent and constitute a well-defined and unified set. All six figures are strictly symmetrical with respect to

[20]Determining the slant of the x is the initial stroke, which is easily recognizable by the deeper color of ink.

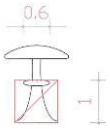
a central vertical axis, and thus all could be made merely with a lathe. Common to each is a base that can be formally fit into an isosceles triangle with its equal sides springing tangentially from the base's sides, and rising elongated upwards as they incline elegantly and symmetrically towards the center .

In the pawn these curves terminate at a height more or less equal to the width of the base, in a small circular element whose diameter is slightly greater than half the width of the base, with a median ratio of around 0.6:1. Taking into account the approximation properly demanded by the difficulty of reading minute dimensions, this conforms to the Golden Ratio (0.618033..../1).[21]

The ratio of the Pawn's total height to the width of its base is less than 2, and on average 1.6:1.

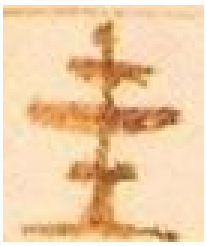
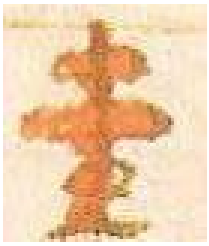
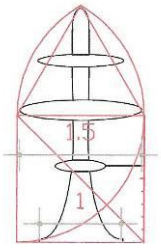
Pawn,   see Pl. 7.

Painted onto all the other figures reduced to a height on average equal to the width of the base, is a small transversal element, more or less elliptical in form, sometimes distinctly oval, with its broadest horizontal axis likewise bearing the ratio of 0.6:1 to the width of the base.

Queen,  Knight    Rook, etc.

In the King, the two lines defining the triangle of the base pass beyond this element and rise parallel to the central axis to define a delicate rectangular body about one millimeter in width, which terminates in a slightly rounded tip.

Across this delicate central body are painted two – in a few cases three[22] – elements of various width, all more or less elliptical, occasionally oval, in form.

King

(The figure of the King can be inscribed into a square surmounted by an equilateral triangle.)

The first from the bottom is noticeably wider than the one above it – approximately 1.5 times the width of the base; its two ends would constitute the top and breadth of a hypothetical square enclosing the area between it and the

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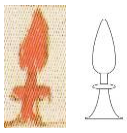
[21] In Euclid's *Elements*, one reads the following definition: "It is said that a segment is divided according to the median when a segment is to the greater part as the greater part is to the lesser one."A_____.C_____B in which the proportion is AB : AC = AC : CB. Extolled as essential, ineffable, unique, unnameable, wonderful, this is the ratio that Luca Pacioli, or more likely Leonardo, called "divine;" familiarity with it is indispensable to the drawing of a regular pentagon and each regular polygon and polyhedron is derived from it. The Golden Ratio is defined, perhaps for the first time, by Kepler as the ratio AB:AC. And designated AB **a**, AC **b**, and CB **c**, **a:b=b:c** as well as **a=b+c**. The Golden Ratio **a/b**, is designated with the letter Φ for the first time by mathematicians Mark Barr and William Schooling, most likely in honor of Pheidias; Theodore Andrea Cook, *The Curves of Life* (Dover Books Explaining Science). The value for Φ is found by expanding the two given equations: Φ=a/b=b/c, Φ=a/Φ/(a-b), Φ =a/Φ/(a-a/Φ), Φ=a/(a Φ-a), aΦ² -a Φ = a, the quadratic equation aΦ² -a Φ-a =0 for a = 1 drawn from Φ = √5/2 + ½ = 1,618033988; this is the Golden Ratio, as defined above.
Its inverse, 1/1,618033988=0,618033988 is the Golden Section, that is, part AC of the segment, given that AB is equal to 1. Since √5/2, according to the Pythagoras' theorem, is the measure of the diagonal of semi-square of side 1, the Golden Section of a given unit can be constructed geometrically by subtracting
from it √5/2, the diagonal of the semi-square of side 1, half the side, precisely ½.

[22] There are only three examples of the King with three as opposed to two diagonal elements: one on fol. 16v and two on fol. 17r. On the folios on which the images of piece are "exclusively painted," as we will see later, these seem to be errors due to the imprecision with which they were drawn (see pl. 10) rather than the result of deliberate choice.

piece's base. The one or two elements inserted between this and the top of the figure can be inscribed within an equilateral triangle, whose base would coincide with the wider transverse element and whose vertex would coincide with the figure's apex.

The Queen appears in two versions. In the first version, above the base, and at some distance from it, is a second element of around the same width, but semicircular, and convex towards the bottom; rising from it, spring two equal lines inflected symmetrically and elegantly inwards and reflecting those of the base below. These terminate in a rounded, semicircular tip.

In the second version, the form of the element above the base resembles a feather or plume that ends in a rounded tip.

 Queen,  Queen.

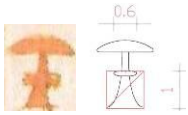
In the case of both the Bishop and Knight, above the first transverse element, which they share with the king and queen, is a sector of a circle, convex towards the top whose span approximates its height with respect to the base.

The Knight can be distinguished from the Bishop because its slender central body reemerges above the sector of the circle. The figure of the Bishop, on the other hand, terminates in this sector.

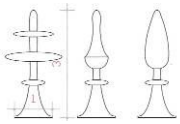
As with the Knight, King, and Queen, so too in the Rook, the slender central body reemerges above the transverse element that distinguishes it. This piece is comprised of an elegant figure that could be inscribed in a low isosceles trapezium, with a height-to-width ratio of approximately 1:6, and whose equal sides are symmetrically inclined towards the central axis while its two horizontals are bowed – the bottom, larger one, upwards, the top, shorter one, downwards.

 Rook

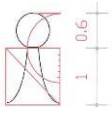
In the Rook, Knight, and Bishop, the ratio of the figure's height to the width of its base is approximately 2:1.

Rook  Knight  Bishop 

In the King and Queen the ratio of height to width is approximately 3:1.

King, Queens 

In the Pawn, as noted, this ratio is 1.6:1.

Pawn,  see Pl. 7.

Given the tiny dimensions of each square in the larger chessboards (e.g. 10.62=85/8 mm; see pg. 24), each figure's baseline is only a few millimeters long and difficult to discern; it generally varies even in terms of the tiny brushstrokes used to define it, and is on average longer in the pawns (6-7 mm) and shorter (4-5 mm) in the other game pieces.

This difference -- pegging the breadth of the base to the height of the represented figures -- together with their insertion in checkerboard squares that literally frame them, creates a sophisticated articulation of a hierarchy based on heights -- the King and Queen being the tallest, the Rook, Bishop, and Knight mid-range, and the Pawns shorter than they are actually depicted. In other words, each game piece in the drawing occupies its square in the largest possible size that can fit inside it.

As already noted, the representations of the pieces in the manuscript often, though not always, differ considerably from the simple schematic representations that would have sufficed to identify the pieces in illustrations of game situations.

Indeed, approaching pictorial representation despite their small format and rapid execution, the images were clearly created by an expert handling of the brush with various degrees of pressure that left faint shadows through the build-up or dilution of the paint and thus bestowed on them a wonderful sense of three-dimensionality.

The differences in execution that we have been noting - first in the rendering of the chessboards, sometimes freehand, sometimes with a ruler, and more so in the various modes of applying paint – open up the possibility of questioning the established attribution of the entire text to the sole hand of Luca Pacioli.

Let us examine these carefully:

In the Pawn, for example, the little disk capping the top is often created by a light, circular brushstroke, which, along with the elegant shading, grants it an effective impression of sphericity and occasionally even of a delicate luminosity at the center created by the re-absorption of paint at the instant when the tip of the brush was lifted off the paper.

 [23]

This impression is accentuated by the heavier deposit of ink towards the base and one side. Of note here – but I will return to this later – is that this deposit always occurs on the right side.

The whole is obtained quite rapidly, with a disregard for slight disparities.

These, however, never render the identity of the represented figure ambiguous.

Careful observation reveals another fundamental difference, namely, that the representation of the pieces, precisely in those areas where the color is brightest, is reinforced by an outline, rapidly executed with the tip of the brush – or, more likely, with a soft pen-nib dipped in light sepia ink – while the inner side of this contour is colored in red or a dark gray.

Notable too is that in all the representations in which this outline is *not* present and in which the figures are drafted solely in color, the paint is uniformly applied and flat.

In addition, the figures rendered without outlines are also often out of proportion to each other, even when appearing on the same page. In them the application of color, besides being minimal, is at times imprecise and sloppy.

In the figures with outlines, however, the color is brighter because it is applied with multiple strokes of the brush.

Noteworthy too is that the latter are always slightly inclined, that is, leaning towards the right – an inclination accentuated, as noted, by the shadow created by the accumulation and reduction of paint and highlights in the rendering of the spherule (see Pl. 9 - 1).

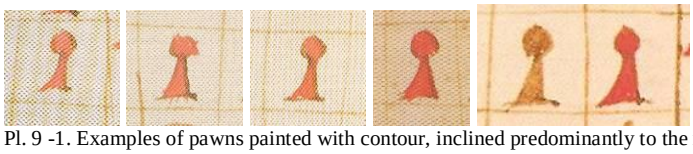
The figures without contours, however, are oriented either vertically or, occasionally, inclined towards the left (see Pl. 9 - 2).

The fact that the chessboards were drawn in two different manners – some free hand, some with the help of a ruler – does not necessarily means that they were made by two draughts men, at least not as long as no account is taken of the other differences already noted and of the relationship between the two different manners of rendering chess pieces and the two different manners of rendering chessboards.

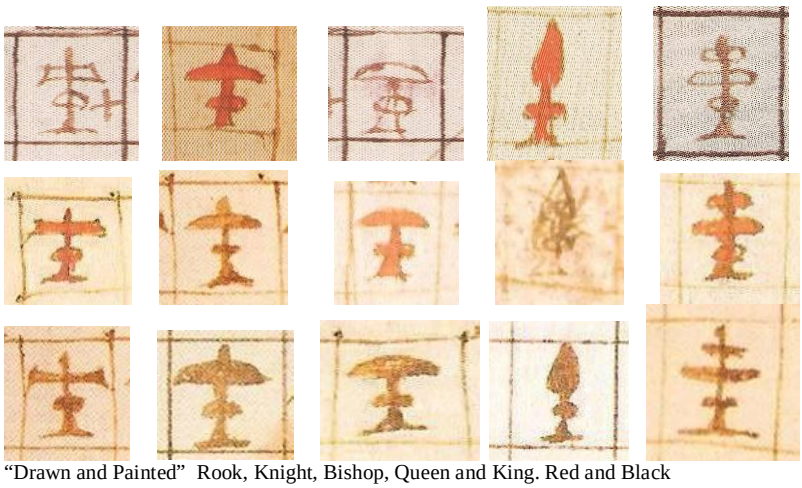
We see, in fact, that with the sole exception of fol. 25r, the two different manners of portraying the pieces, which for the sake of convenience I will distinguish as "**drawn and painted**" and "**exclusively painted**," can be classified into **three** well-defined and identifiable **groups** in the manuscript. (Pl. 10).

[23] Inserted here is an example of a pawn from folio 3r, whose dimensions are approximately those of the manuscript. What was observed directly in the original is here still legible even though this image was taken from a second or third-hand reproduction. It is more difficult to see the contour described below. For details of such, see Pl. 9.

PLATE 9.

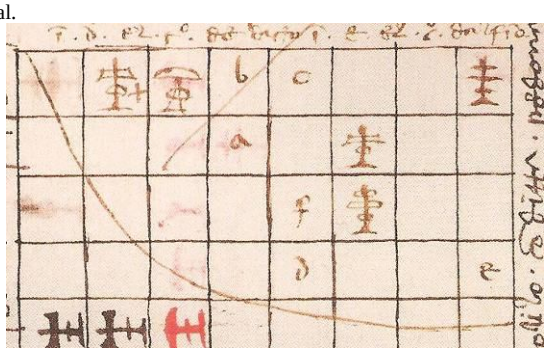


Pl. 9 -1. Examples of pawns painted with contour, inclined predominantly to the right.



"Drawn and Painted" Rook, Knight, Bishop, Queen and King. Red and Black
In the reproductions of the "black" pieces, the stroke of the outline is difficult to read. Nonetheless It is always present and legible in the original.

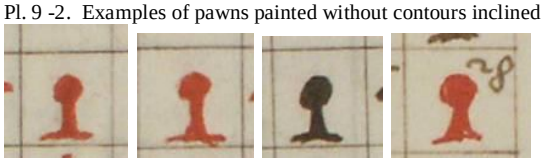
Folio 1r., a King and a Queen on in order to be repositioned are deleted with left-handed strokes typical of Leonardo (see Pl. 11 further down).



Folio 25r Detail (match no. 60) The precise and neat drawing of these uncolored figures evidences the mentioned inclination on the right

The juxtaposition of "drawn and painted" figures and the same figures merely drawn, but not colored, underlines that are doubtlessly by the same hand.

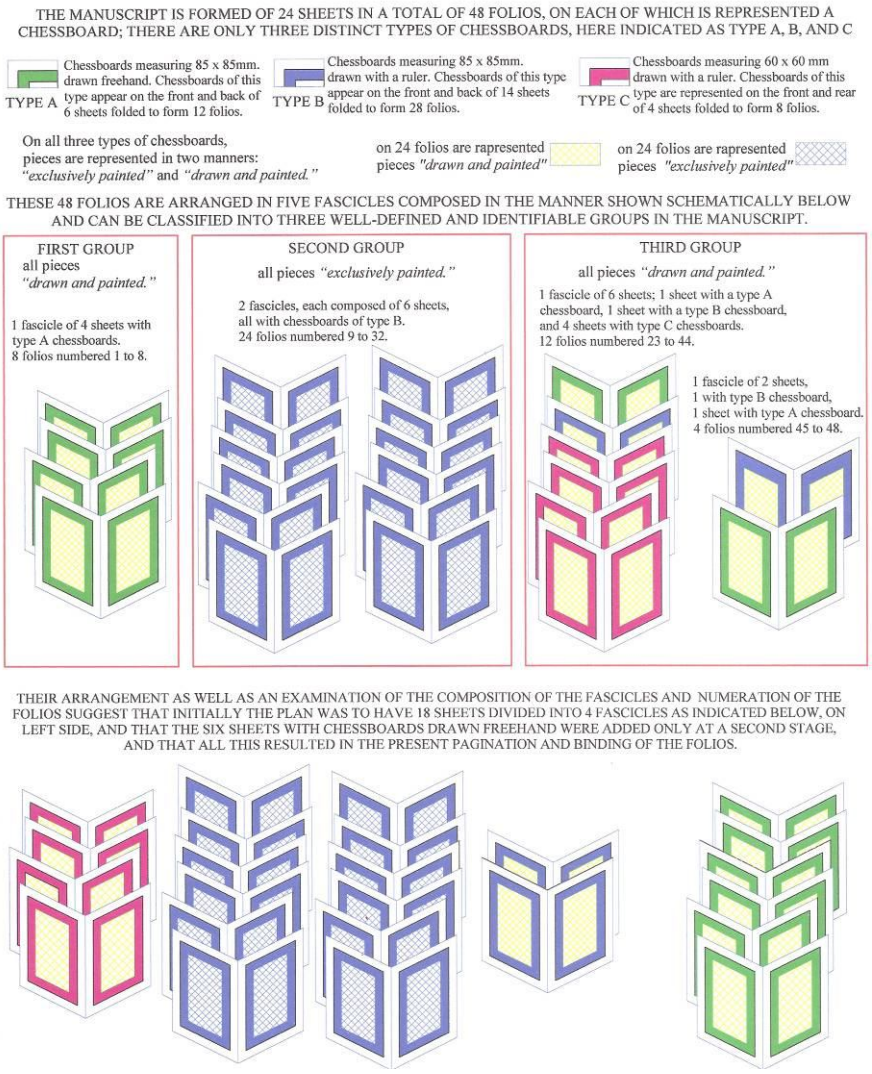
Pl. 9 -2. Examples of pawns painted without contours inclined predominantly to the left.



"Exclusively painted" Rook, Knight, Bishop, Queen, and King. Red and black.

Highly obvious in each case is the difference of the hand that rendered the "drawn and painted" pieces from the one that rendered the "exclusively painted" ones.
The first are executed with a more expressive line with an accentuated shadow due to the accumulation of color at the sides; the second are neutral with uniform application of color, so that the figures appear "flat." The first incline predominantly to the right, while the second are perfectly vertical, or occasionally incline slightly to the left. The ink used is likewise different; in the "drawn and painted" pieces, the red is brighter and black much lighter with respect to the color of the "exclusively painted" pieces. Even the fact that the Queen is represented in two manners – in the form of a plume in the first, and the form of a fountain support in the second – confirms that we are looking at two draftsmen.

PLATE 10.



Only "drawn and painted" pieces are depicted **in the first sixteen consecutive pages** of the manuscript, that is, fols. 1 to 8, which contain nineteen matches – three more than sixteen because three chessboards (fols. 1r, 1v. and 4r, respectively) simultaneously display two matches, which are separated from each other by a double line, and can be read by rotating the page 180º. In this initial section, the pieces are likewise exclusively depicted on chessboards drawn freehand.

In the central portion of the manuscript, from fols. 9 - 32 (inclusive), fifty-seven matches are represented on forty-eight successive page faces. Here too the nine additional ones (57-48) owe their existence to the fact that two matches are shown on nine of the chessboards (fols. 9r., 9v., 14r., 17r., 17v., 22r., 23r., 25r., and 26r). With the sole exception of fol. 25r, already noted and examined below, the pieces in these fifty-seven matches are all "exclusively painted" and appear on chessboards drawn with a ruler.

Finally, **in the third and final section**, from fols. 33 - 48, in which thirty-eight matches are represented on thirty-two page faces (six more than thirty two since two matches are depicted on fols. 33v, 34v, 36v, 39v, 42v and 47r), the pieces are "drawn and painted," while the chessboards are sometimes drawn in freehand, sometimes with a ruler.

Analyzing these three sections, we may deduce that the two modes of representing the pieces appear with nearly equal frequency; 57-1=**56** matches are **"exclusively painted",** and 19+38+1=**58** are **"drawn and painted."** As we shall see, the match on fol. 25r must be attributed to the same hand that drew the "drawn and painted" pieces.

In addition, in the first two sections, the manner of representing the pieces is consistent with the manner of drawing the chessboards; the "drawn and painted" pieces appear on chessboards drawn freehand while the "exclusively painted"

pieces appear on chessboards drawn with a ruler. In the third section, however, the "drawn and painted" pieces appear on chessboards drawn either freehand or with a ruler.

This is an inconsistency that finds a logical explanation if two other parameters are carefully examined:

The rules of the game and the composition of the fascicles.

The rules of the game:

Necessary to bear in mind is that the matches illustrated in the manuscript are being played in two different modes: according to older medieval rules, what Pacioli only in very few case refers to as *"dritta"* or *"alla dritta"*, and according to the newest ones what Pacioli indiscriminately always refers to as *"rabio"* or *"a la rabiosa."*

The composition of the fascicles:

Indeed a study of the book's binding reveals that for a long time before it was bound, there were five fascicles[24] composed in the order in which they are rebound today. One consists of four sheets of paper (a quaternion), three of six sheets (sexternions), and one of two sheets (a duernion). Each sheet of paper measures 150 x 220 mm, and was folded in half to form two folios of 150 x 110 mm. (Pl. 10).

The first part coincides with the first fascicle and consists entirely of matches played according to the new rules. Here, as noted, there are only matches *"a la rabiosa"*[25] and pieces "drawn and painted" on chessboards drawn in freehand.

The central portion of the manuscript coincides with the first two sexternions.

The first of these comprises fols. 9-20 and contains solely matches played according to the old rules, illustrated with pieces "exclusively painted" on chessboards drawn with a ruler. The second comprises fols. 21-32 and likewise contains only matches played according to the old rules, which are represented on chessboards drawn with a ruler and pieces "exclusively painted," with the already noted exception of fol. 25r, where, in match no. 60, the pieces are solely drawn but not colored.

The third section contains the third sexternion, which begins with fol. 33 and concludes with fol. 44, which are parts of the same sheet. Depicted on this sheet are five matches of which four are *"rabio"*, and one, on fol. 33v, is *"alla dritta"* and shares the chessboard with a rabio match. On these four pages all the pieces are "drawn and painted" and the four chessboards are drawn freehand. In this same third sexternion, on the second (no. 34) and last folio (no. 43), which are parts of the same sheet, appear five matches played with the old rules; their pieces are "drawn and painted" on four chessboards drawn with a ruler.

All the other chessboards on the remaining sheets inserted between fols. 34 and 43 are also drawn with a ruler, but with the difference that the length of their sides has been reduced to ca. 60 mm. On these smaller chessboards, only partly drawn with a ruler, as we have noted, the pieces are "drawn and painted," though on a smaller scale. The black ones, in particular, have contours that are often illegible. The matches represented are all of the type played with the old rules.

Thus too in the final duernion, we distinguish fols. 46 and 47 of the manuscript's inner sheets, whose four pages contain four matches *"a la rabiosa"* and one *"alla dritta"* (which shares fol. 47r with a *"rabio"* match), and in which the pieces are "drawn and painted" on chessboards drawn in freehand. Meanwhile, on the outer sheet, whose folds form fols. 45 and 48, the pieces, still of the "drawn and painted" type, are represented on chessboards drawn with a ruler. The two matches on fols. 45r and 45v are of the *"rabio"* type, while the two on 48r and 48v are played according to the old rules.

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[24] It is clear that the manuscript consists of five fascicles even without disassembling it into the 24 sheets that constitute it; the pages opening and concluding each fascicle have stains caused by humidity and wear that do not occur on the inner pages. On this point, see GLI SCACCHI DI LUCA PACIOLI– EVOLUZIONE RINASCIMENTALE DI UN GIOCO MATEMATICO, Aboca Museum, Borgo S. Sepolcro (AR) I.

[25] Actually there are also three problems played by the old rules: two entitled "*alla dritta*" (one way of defining the old rules), and one left without a title. All three, however, are inserted upside-down (to read them one needs to rotate the page 180º) on a chessboard on which is inscribed a problem entitled "*rabio*." This creates the impression of a fascicle devoted to the exposition of problems played according to modern rules.

Based on this analysis we conclude that the "exclusively painted" pieces were, without exception, used to illustrate matches played with the old rules. There are 56 such matches displayed on 48 chessboards, all of the same larger size, on the 48 consecutive pages (fols. 9-32) of the second and third fascicles of six sheets apiece.

Meanwhile, "drawn and painted" pieces illustrate matches played with either the old or the new rules.

That is, the first matches played with the old rules are represented on pages prepared with ruler-drawn chessboards. Of these there are 26: nineteen on the eight consecutive central folios (35 to 42) of the fourth fascicle of six sheets that display sixteen, 60 mm-square ruler-drawn chessboards, and an additional seven matches on the four folios with the four larger ruler-drawn chessboards on fols. 34 and 43 and fols. 45 and 48 - the second and next to last ones of the fourth fascicle and the first and last ones of the fifth and final fascicle.

There are two exceptions. The first consists of five matches played with the old rules that are represented with "drawn and painted" pieces on chessboards drawn freehand. Their peculiarity is underscored by two facts: 1) these two matches are called *"dritta"* or *"alla dritta,"* terms that never appear in other matches played with the same rules; 2) they share a chessboard with a match played according to the modern rules, which are always referred to as *"rabio"* or *"alla rabiosa."* The second exception lies in a match on folio 25r. with chess pieces drawn by the same hand that drew all the other "drawn and painted" pieces, and which is also the only one drawn but not painted.

In toto, the matches played with the old rules and illustrated with "drawn and painted" chess pieces amount to 32 (26+5+1).

The matches played with the new rules and depicted with "drawn and painted" pieces, on the other hand, are illustrated on chessboards drawn freehand.

There are 24 such matches and they appear on the first 16 consecutive pages, from fols. 1 to 8, of the first four- sheets fascicle; on four pages – fols. 33 and 44 – of the fourth fascicle of six sheets; and four pages – fols. 46 and 47 – of the fifth and final fascicle of two sheets. Notable too is that there are two further *"alla rabiosa"* matches with "drawn and painted" pieces on two ruler-drawn chessboards on fols. 45r and 45v. [26]

Thus out of all the matches played with modern rules, *"rabio"* or *"alla rabiosa"* are 26 and are all represented with "drawn and painted" chess pieces.

To conclude, if the binding is undone, the manuscript becomes divisible into two parts of twelve sheets apiece:

- a first section, in which only "exclusively painted" pieces are used to illustrate matches played with old rules on ruler-drawn chessboards of the larger size.
- a second, containing only "drawn and painted" pieces in illustrations of games played either with the old rules and represented on ruler-drawn chessboards of a larger or smaller size, or with the modern ones and represented only on chessboards drawn exclusively freehand.

This second section is further divisible in two:

- a first part, in which the matches played with the old rules are illustrated with "drawn and painted" pieces on folios previously prepared with ruler-drawn chessboards.
- a second, in which the matches played with modern rules are illustrated with "drawn and painted" pieces on chessboards executed freehand.

Disregarding the current order of the folios' pagination, one can imagine an initial situation in which, prior to the formulation of the manuscript, on which known and annotated matches in various fascicles were designated as *"meis"* or *"nostris,"* the sheets with the chessboards were divided into two sections:

[26] At the end of this chapter I note the opinion of Master José Antonio Garzón, who claims that of the 26 problems that follow the modern rules, twenty four are original while two were already known at the time. I do not have the expertise to examine the matches from this angle. Nevertheless the subdivision revealed below could have been based on this valuation. That is, we have twenty-four completely original matches "*alla rabiosa*" on 24 chessboards drawn freehand and two familiar matches *"alla rabiosa"* on chessboards drawn in advance with the help of a ruler.

- A first part, consisting of twelve sheets (two sexternions), all prepared front and back with drawings of 48, 85-mm square chessboards.
- A second, comprised of six sheets (a quaternion), that is, four sheets (a duernion) prepared front and back with a schematic grid depicting sixteen 60-mm.-square chessboards, plus two sheets prepared front and rear with drawings of eight 85-mm.-square chessboards.

To these second six sheets were added another six (a sexternion) on which were drawn an additional twenty-four chessboards, all freehand. This addition could have been made at a second stage for the purpose of displaying twenty-four novel problems played by the new rules "alla rabiosa," which, as we recall, were being developed at the time. See page 9.

As we have seen, this subdivision was transposed into the manuscript in such a manner that the 24 folios created from the first twelve sheets were all assembled in the second and third fascicle, each with six sheets. These were all numbered in order from fols. 9 through 32, while the twenty-four folios derived from the second twelve sheets were added partly to the beginning and partly to the end of the manuscript.

Some good reason must have informed this arrangement.

Given, as we have seen, that the pagination of the folios was noted in an orderly manner only in the second and third sexternions while in all the others it was added either after the texts were written or embedded within them, and given that the second sexternion begins with the number 9, the manuscript may have been meant to begin with a fascicle of four sheets.

Four sheets with 60-mm-square chessboards were, in fact prepared. Then, at the moment when the fascicles were sewn together, Pacioli preferred to begin with the four sheets drawn from the six with freehand-drawn chessboards, most likely because he deemed them more interesting – perhaps because these depicted matches played with the modern rules, and particularly because, a match played by the new rules as well as one played by the old ones were illustrated on the same chessboard on both the recto and verso of fol. 1.

One can thus assume greater interest in the *"rabio"* or *"alla rabiosa"* matches that appear on the chessboards drawn in freehand – an appreciation that could be attributed as much to the intelligence of whoever invented the matches played with the new rules as to the beauty of the pieces and the elegance with which they were rendered, drawn, and colored, and which, despite their tiny size, nonetheless displayed certain pictorial effects that are still evident today, 500 years later.

To conclude, the two modes of representing the figures reveal two different personalities. Of these, the one whose task was facilitated by the simple stylization of the game pieces, drew only familiar matches played by the old rules on pre-drawn chessboards without any particular skill or inventiveness, while the other depicted both old, well known matches and those played according to new rules (24 out of 26 of which were most probably newly conceived – see p. 33) either on prepared chessboards or on ones drawn freehand with great speed and mastery with the decisive and fresh stroke of a person pinning down an idea at the moment of its conception.

The difference between them is more evident wherever the draftsman of the "drawn and painted" pieces merely drew them and omitted the color, sometimes because the color was irrelevant to the comprehension of the match depicted, (Pl. 11), sometimes because the figure in the square was deleted before being colored and redrawn in a neighboring square.

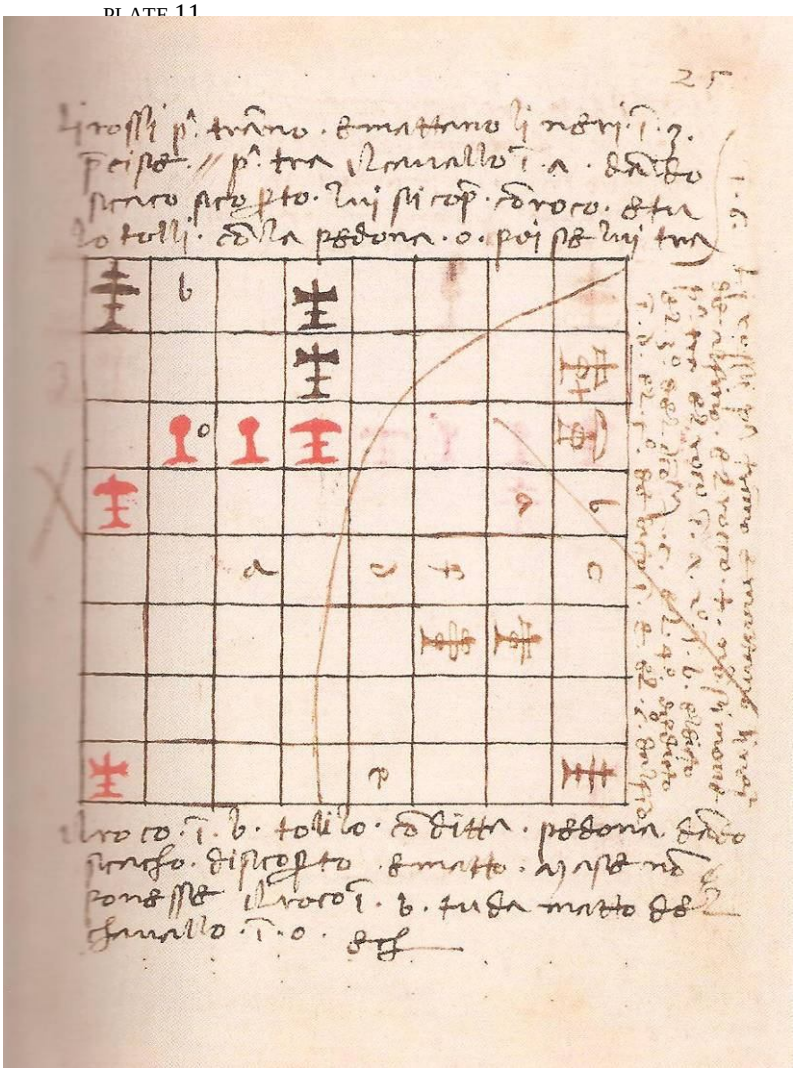
That the drawing of these figures is by the same hand as the one that "drew and painted" them is obvious from the calligraphic precision of the mark, which is rendered more legible by the lack of subsequent coloring, clearly showing that it had been executed rapidly and perfectly without hesitation -- as (if one looks closely) were the figures painted later, (Pl. 9-1).

The figures that are merely drawn are few in number but highly significant.

In particular, as already noted, there is match no. 60^{27}, on fol. 25r, in which all the pieces were merely outlined, (Pl. 11). As well the figures of one King and one Queen in match no. 2, on fol. 1r. Two other figures – that of a Rook

in match no. 6 on fol. 2v. and a King and a Queen in match no. 105 on fol. 44v. crossed out lightly with an X or with two short strokes extending from bottom right to top left.

PLATE 11



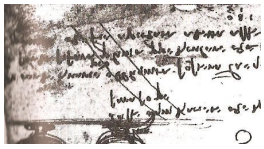
Folio 25r . The oft-cited match no. 60 is set on the chessboard drawn on fol. 25r to the side of match no. 59 with "exclusively painted" pieces; its long description occupies the upper portion of the lower margins of the folio above and below the chessboard. The description of match 60 is therefore forcibly squeezed into the right margin of the folio; to read it one must turn the page 90º counter-clockwise. Pacioli penned it in an ink of a different consistency, much lighter than the one used in the description of match no.59.

The aforesaid tiny "x" and crossings out are deletions made by a left-handed person identical to those in Leonardo's folios. Although they are extremely rare among his drawings they are frequent in his writings, a common result of rapid writing in which gesture anticipates thought. The small x used for deletion, which replaces the "line" in thirteen cases and, like it, is executed by a left-handed writer, is typical of Leonardo (Pl. 12). [28]

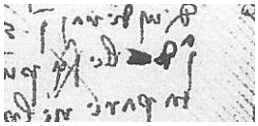
Finally in all these figures the line is continuous and calligraphic, typical of someone who expresses himself through drawing, that is, of someone who regards drawing as writing. Leonardo was just such a person. Many of his folios – *especially 12692r and v of the Rebus folios in the Royal Library at Windsor Castle* – are full of tiny drawings that are equally significant despite their minute dimensions and obvious speed of execution (Pl. 13).

[28] Leonardo read and confirmed the correctness of only these 13 texts, deleting them with the little x. By contrast all 101 texts deleted by Pacioli can be explained simply by the fact that although the work, as we have seen, was carried out in nearly equal parts by both, it was the initiative of Pacioli, who assumed greater responsibility in the revision and correction of the text. Or – but this is a claim that goes beyond my ability to prove – by the possibility that these 13 problems involved particular technical features that Pacioli asked Leonardo to check.

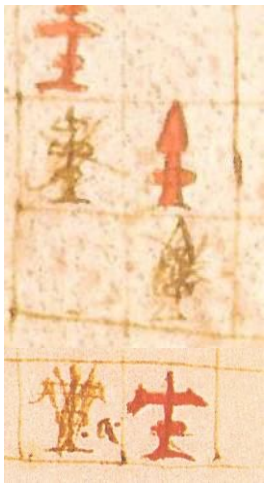
PLATE 12.

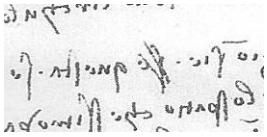


Codex Atlanticus, fol. 400 v.d.



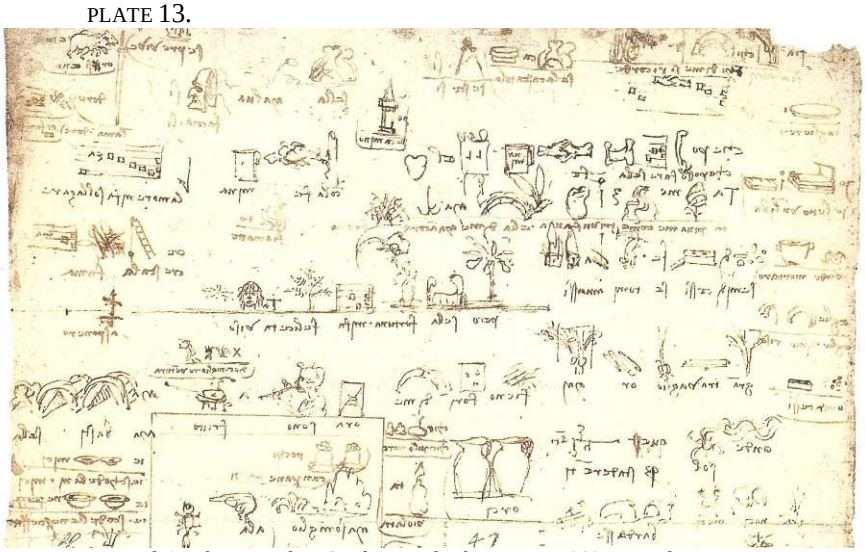
Folio 14r Ms. B 2173, Paris, Bibliothèque National de France





Folio 14r Ms. B 2173, Paris, Bibliothèque National de France

PLATE 13.



Leonardo's rebus, Windsor Castle, Royal Library RL 129692. Detail.

In the series of tiny drawings illustrating the rebus, we may note the same unwavering decisiveness that went into drawing the figures of chess pieces "with contours" in the manuscript.

The slope towards the right observed in the figures of the manuscript – however minimal – is noticeable in these small drawings as well.

For example, at the center of the section of the folio reproduced here, beneath the rebus, which in Leonardo's handwriting naturally reads from right to left, lie the words *ora sono fritto,* in which the word "ora" – hour - is represented with an hourglass, in which we may recognize the aforementioned slope both in the two bells, and at the side, in the two monsters, as well as in the hourglass itself.

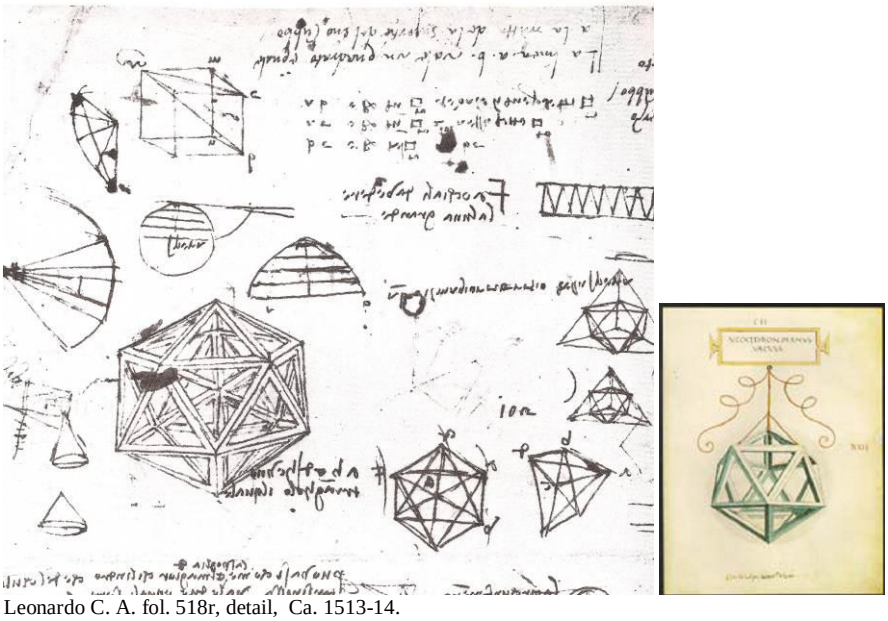
Higher up on the folio, the little belltower is inclined towards the right.

It is true that in these tiny drawings, as in those in the margins of the codex on the flight of birds in the Biblioteca Reale of Turin, Leonardo nearly always used short but important hatching strokes to emphasize volume, whereas the contours of the chess pieces contain no hatching at all. However, account must be taken of the fact that in these drawings as in actuality the figures had to be colored in order to make the two sets distinguishable. As we have seen, it is precisely through color that Leonardo addressed the challenge of capturing the illusion of relief -- and whenever he did use color, he did not use hatching.

Some scholars have pointed out to me that the drawings exhibit none of the grace of ones solidly attributed to Leonardo.

Professor Pietro Marani in particular considers them *poor and insignificant*, a harsh judgment passed without direct knowledge of the original, and most likely without any account of the fact that these were folios full of notes meant for personal use, in which, as in so many others, Leonardo paid absolutely no attention to the aesthetics of his stroke (Pl. 14).

PLATE 14.



Leonardo C. A. fol. 518r, detail, Ca. 1513-14.
Among the many examples of folios in which Leonardo made drawings with no particular attention or aesthetic sense, but solely to note down an idea, I chose this one where among various geometric schemes, he drew within an empty icosahedron a sort of structure bearing an external framework by following the axes that converge at the center from the vertices of the two opposite sides. Here it is evident how far we are from the elegant figure in DE DIVINA PROPORZIONE (at the side). Leonardo seems much more interested in fixing the geometric correspondence among the internal and external triangles of the geometric figure that seems very hastily drawn, merely as a memory cue.

Equally true is that it is impossible to determine whether the chessboards drawn freehand were executed by a right or left hand because when rotated 180º – a maneuver necessary to read the text wherever two matches are represented simultaneously on the same chessboard– the horizontal lines are reversible and could have been executed either from right to left, or vice versa. The same does not hold true for diagonal lines, however, which when used for deletions retain an identical slant when the page is turned upside down.

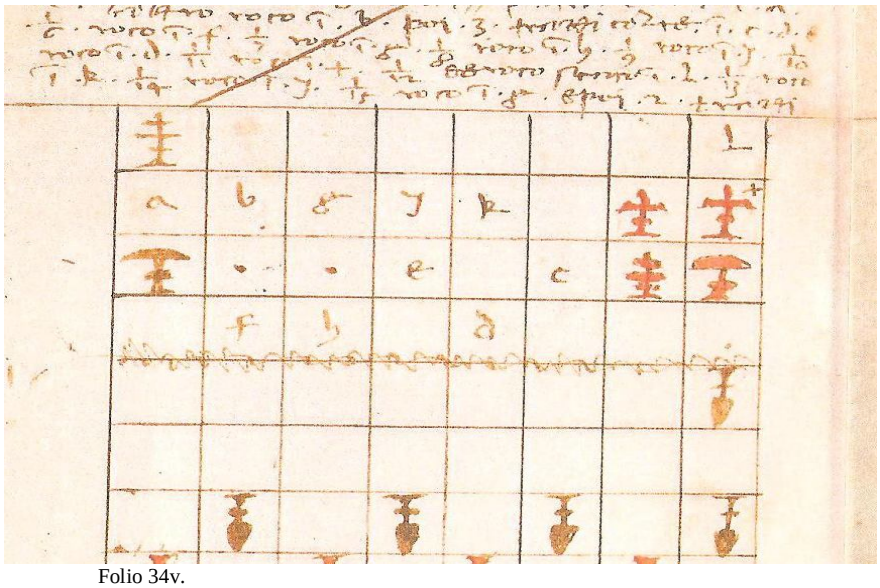
That the draftsman was left-handed is also evident from an observation of the drawings' already noted inclination to the right, and is further supported by the mark that separates match no. 81 from 82 on fol. 34v, where the horizontal median of the chessboard is emphasized in order to serve as a border between two matches with a continuous, undulating line inclined towards the left, which could only have been drawn by a left-handed person (Pl. 15).

Insignificant in itself too is the fact that the ciphers written at the side of each chessboard were all made from left to right because Leonardo generally wrote numerals from left to right when inserting them at the side of notes made in his characteristic mirror calligraphy. Perhaps some significance can be attached to an 01 written in place of a 10 in match no. 103 on fol. 43v – a match, of course, with "drawn and painted" pieces.

Yet what can be said about the pieces that were "exclusively painted" as opposed to the "drawn and painted" ones these were executed diligently albeit a bit sloppily, with care but little understanding? Notable too is that, save for a few "botched" figures that were repainted, they contain no after thoughts or deletion marks and thus reflect a manner typical of someone copying rather than creating.

In all 56 of these matches, there is only one error in positioning; on fol. 31v, a red king is crossed out and repainted in a neighboring square. In this case, the deletion marks were done with a right hand: a few short strokes from bottom left to top right (Pl. 9 -2).

PLATE 15.



Folio 34v.

These observations lead us to conclude that:

- Pacioli reserved for himself the task of reporting matches played according to the old rules on 48 pages of the manuscript, on which chessboards had been pre-drawn with a ruler.
- Leonardo had a direct hand in the drafting of the manuscript and illustrated 58 out of 114 matches. He reserved for himself those sections of the manuscript that brought together matches played by the new rules, which Luca referred to as *"rabio"* or *"alla rabiosa"* in his writings, and drew chessboards freehand on twenty-four pages.
- Leonardo, however, also illustrated thirty-two matches played with the old rules, on other pages where chessboards had already been prepared with a ruler.
- In one particular case - "match" no. 60 on fol. 25v - he did so almost impulsively, inserting a new problem on a chessboard, where he discerned a suitable place among the situations drawn by Pacioli. A "match" so clear in its unprecedented formulation that the author deemed it unnecessary to distinguish the pieces by color!

An assessment of the extent to which these conclusions affected the evolution of the rules of chess lies beyond my competence.

I thus limit myself to quoting Josè A. Garzòn, who notes: “My opinion is that Pacioli's book.. contains twenty-four original "*a la rabiosa*" chess problems” (though I would argue that it is difficult to credit Pacioli for these), and Sir Raymond Keene, who observes that:

“….there is the alluring possibility that Leonardo himself composed the problem. The possibility that Leonardo did compose this puzzle is enticing and by no means impossible....Only a powerful intelligence could have devised the puzzle and the solution, which would tax the mental powers of most strong players even today and in its complexity and richness could only really be solved easily by a computer. The evidence of a commanding intellect behind this chessboard conundrum is palpable indeed”.

Let us turn back to the images of the figures, which despite their differences, always appear clearly traceable to the same model.

The impression one gets is not that these differences are the product of different manners of drawing the same letter, the complex uniqueness of a fixed graphic symbol, but are rather the logical result of a person, who, having a model before him, adapts it to meet his own capacity and sensibility to the space available.

The model, recognizable despite the inevitable distortions that occur when it is reduced to a synthetic sketch is analogous to a rapid script set in an available space. In and of itself it is an absolute invention; novel in its time, it is equally original today.

The result of this analysis demonstrates that not only are the figures of the chess game pieces represented in Luca Pacioli's manuscript utterly original, but also that Leonardo is their likeliest inventor.

Such an attribution is supported by the expert simplicity with which the set is constructed, its synthesis of form and expression.

It is further corroborated by the geometrical ratios used, which can be traced via historical context to Leonardo's sophisticated and personal study of proportions and forms that could be inscribed in regular polyhedrons or derived thereof.

Finally it is reinforced by the drawings in the totally new sections of the manuscript, in the first and final fascicles. These depict 19 matches and seven additional ones *"alla rabiosa."*

These are drawings and invented situations drafted with a speed, originality, and freshness of invention that jump out at us now that they have been placed in the context of the complex and laborious description of the other 56 "matches" catalogued by Pacioli.

Just as the father begets a son, the son another son, each in his own likeness, thus in the Golden Section when the greater part is united with the entire segment, the resulting sum replaces the entire segment and this resulting sum becomes the greater part" (Kepler).

Let us now turn to the three-dimensional images in Pl. 17. The most complicated task in analyzing the pieces lies in demonstrating the relationships between their heights and deciding which proportions to choose from among the many that could be extrapolated from the depictions in the manuscript for a potentially equally accurate reading.

In dealing with the difficulty of interpreting a manuscript made exclusively for personal use -- a preparatory draft for a version that would have been adapted for print and provided with finely painted illustrations, which would have left no doubts regarding the pieces' forms -- we are privileging the intellectual concept of the work rather than its existent form.

Had DE LUDO SCACHORUM reached the stage of publication, it would have been dedicated, as Pacioli himself noted in DE VIRIBUS QUANTITATIS, to Isabella d'Este. The Marquise was a highly sophisticated person, certainly not inferior in this respect to Duke Ludovico or Galeazzo di Sanseverino, to whom Luca Pacioli dedicated his elegant first work in Milan. This too, as we may recall, had been furnished and completed with drawings by Leonardo.

In any case, the drawings in Pl. 16 mark the end of a process of formal definition, aspects of which (and sometimes only marginal ones) adhere to no precise rules, this being a decisive, albeit partly intuitive synthesis. Choices require a process of sifting through possible options, and in the end privileging one above the rest. After a meticulous comparison of so many gradations between measurements and reliefs, and between the various hypotheses drawn, I have taken the formal coherence of all the pieces as a guide and basis for prioritizing. That is, I have taken as my point of departure the idea that the form of each piece was defined by its measurements, with priority placed above all on its relationship to the others. I therefore begin by assessing the drawings of the six pieces as a whole.

These are revolutionary when compared to chess pieces of the late fifteenth century, yet they do remain specimens of their time and of the forms desired and appreciated in the years when the manuscript was planned, finalized, and drawn with the intention of being published and dedicated to the Lords of Mantua.

To our eyes they may seem ahead of their time, almost futuristic, a sense we get from so many of Leonardo's inventions. Yet to his contemporaries, to cultivated people – above all to Luca Pacioli –, they would have appeared classical, ancient. Indeed in reality they are the sophisticated product of an era and taste that certainly looked at antiquity with fresh and remarkably creative eyes, but also studied the classical world of the Greeks and Romans. It was in this world that they could dote on a unique fusion of ideas, constantly renewed and sustained by an endless, enthusiastic, and inexhaustible supply of literary and formal rediscoveries, seek confirmation of their own new manner of feeling,[29] and find ideal reasons, philosophical motives, and stylistic elements to unleash themselves from the recent past.

Yet, as Alois Riegl pointed out,[30] every form is born of a pre-existent one, just as a word is not invented, but may come into being solely through the evolution, corruption, or synthesis of one or more existing words. A new form is not conceived out of nothing even in the case of Leonardo. In this period, men of great sensibility and culture, such as Leonardo, Luca or Donato Bramante, paid heed to the luminous examples of Brunelleschi, Piero, Donatello, Verrocchio, but above all to Leon Battista Alberti and his study of the Greek and Roman world based on a new aesthetic that all of them were determined to follow.

[29] It is certainly in this sense, moreover, that one must read Lorenzo the Magnificent's motto *"il temps revient."*

[30] Alois Riegl, SPÄTRÖMISCHE KUNSTINDUSTRIE, Vienna, 1901. Italian edition, ARTE TARDO ROMANA, Turin: Giulio Einaudi, S. p. A.

Alberti's great lesson, which inspired not only architecture but all the figurative arts, could be drawn more from the Tempio Malatestiano and the Palazzo Rucellai than from the completion of Santa Maria Novella. The practical, visible lesson of these masterpieces – especially of the first – differ from those drawn from history, literature, or theory - differ in the way that a verbal language differs from that of forms, which has been equally used from time immemorial to express desires, concepts, and modes of existence.

Within this world of forms, there was also what we would now call "contamination" – precisely that which had no place in DE RE AEDIFICATORIA. It was by building rather than theorizing that Leon Battista Alberti communicated, and through an example, albeit unfinished, that he offered a new, modern lesson in the unconventional use of formal elements born from and codified by precise functions that had been adapted to meet new and different demands: ancient forms with new functions. Adhering to the precise rules of modularity, proportion, and symmetry, he approached various architectonic elements with much liberty and a new aesthetic, and -- after the long interlude of the Middle Ages – once again restored the centrality of the human figure. In this same period, Greek and Roman antiquities were sought, discovered, and studied, and inspired every kind of form – from the majestic circular dome and the grand arch of triumph to the tiniest architectural or decorative detail.

If in the aggregate therefore the Pacioli chess pieces recall the antique, it is in their details that they betray the forms that inspired them. Of course their prototypes cannot be discovered in excavations as there were no chess pieces in antiquity. It is thus in another specific decorative feature of the classical world – studied, loved, and imitated – that we can find the stylistic element that inspired Leonardo.

We have seen that with the Pawn, Leonardo took up the best known and most widespread form of his time and re-designed it with new proportions, using the Golden Ratio for the relationship between the taller and more slender base and the spherical element crowning it. The Queen's base is similar to that of the Pawn. In her height, however, she is the only one of the six pieces to be represented in two different forms, as already noted. We find hints of both these forms in drawings by Leonardo, especially in one that builds on the form of a delicate fountain base.[31] Meanwhile we find the uppermost section -- which we have referred to either as "feather-" or "plume-shaped"– sketched in the aforementioned folio of *De rebus*. Here it basically repeats the silhouette of the cypresses painted in the background of the *Annunciation* (Pl. 16). Thus from a stylistic and formal point of view, these two forms of queen may be considered equivalent in the sense that both derive from Leonardo.

Yet the two Queen types are not absolute equals with respect to their frequency and distribution in the pages of the manuscript. In fact there are 65 representations (67 if we factor in two Queens that were redrawn when repositioned) of the small "cypress-shaped" Queen as opposed to 21 of her "fountain base" counterpart. And whereas the first appears only on the 58 chessboards in which the pieces were "drawn and painted," the second is depicted only on the 56 where they were "exclusively painted."

PLATE 16

Comparison between the two manners of representing the Queen and images of cypresses from the background of the *Annunciation*, or details from the Rebus folio (see above) on the left, and with the drawing of a fountain base, c. 1497-1500, Ms. I Madrid, fols. 115r., on the right.

[31] In studies and drawings of fountains, ca. 1587-90, Codex Atlanticus, fols. 293r-b and 212r-a, and ca. 1497-1500. Ms. I, Madrid; in C. Pedretti, LEONARDO ARCHITETTO, p. 311.

Naturally in a set of pieces used for the game, there had to be a single type of Queen. Wishing to derive from the drawings of the manuscript a coherent set most likely designed by Leonardo, I opted for the small cypress-shaped Queen.

PLATE 17

Three-dimensional reconstruction of the complete series.

My choice does not refute the possibility that he invented the other one as well, but it underscores his preference for this one in the 58 problems that he drew with his own hand. In the real set that Leonardo and Luca Pacioli used to position the pieces on the board, the Queen was probably of the "fountain base" type, which Luca depicted when drawing the pieces on the chessboards in the manuscript. Meanwhile Leonardo, for some reason of his own, retained the forms of this set only in the Pawn, King, Rook, Knight, and Bishop, but changed it for the Queen.

The form of the King, seemingly an elaboration of the Bishop's, seems utterly original. Likewise the figures of the Knight and the Rook, though skillfully adapted to that of the Pawn, conform well to the stylistic unity of the whole set, drawing essentially from the figure of the Bishop, the most basic of all three.

All these appear novel, having nothing in common with the forms of known game pieces of the time. Yet as actual forms, they are not altogether new. In fact, the thin, streamlined body that elegantly supports the large and delicate sphere, recalls forms in certain friezes painted on architectural responds in the Fourth Pompeian Style, which inspired the decoration of the Domus Aurea, rediscovered precisely in the late fifteenth century at the initiative of artists, contemporaries, and friends of Leonardo,[32] who lowered themselves from the open excavations on the Oppio Colle into the vaults of Nero's house.

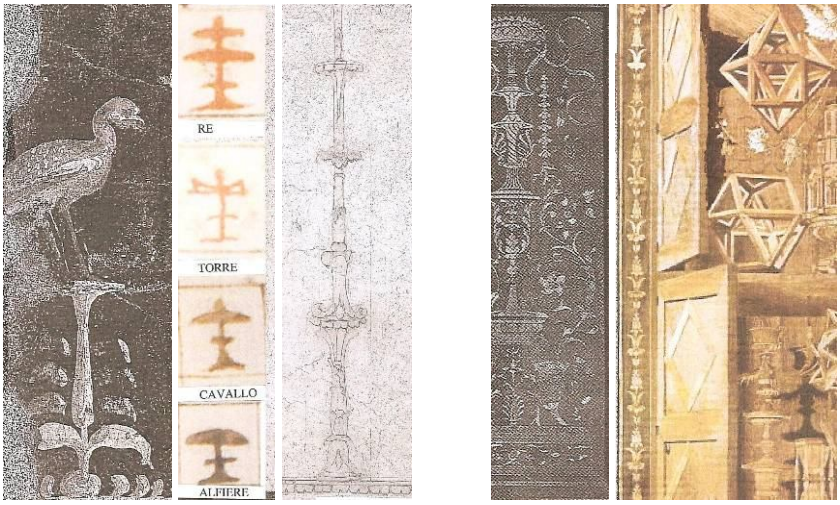
Perhaps it was the image of the new Pawn, even if based on the one already in use, that suggested the adoption of the aforementioned form for the Bishop, Knight, Rook, and King. Or perhaps the other pieces inspired the re-proportioning of the Pawn's figure. The fact is that the six slender and elegant chess pieces conceived as a whole by Leonardo for Luca Pacioli's treatise call to mind the refined ornaments and candelabra that appear for the first time -- and precisely in these years -- in the intarsia of Giovanni da Verona – forms that in their aggregate recreated stylistically those that were discovered and called "grottesche" based on the site in which they had been seen (Pl. 18).

As so often happened at the time, the form of an ornament used in the Roman period was appropriated and transformed with refined humanist sensibility and used for aesthetic ends in a different object with a new and dissimilar function. In this case too, it was born not in a vacuum but from ancient forms after a long hiatus. These forms lay greatest emphasis on features that we have already observed in the painted pieces of the manuscript. Conspicuous above all is the Queen, who is unique among the pieces illustrated on the chessboards. She stands out due to her verticality, and thus recalls with even greater clarity the fountain mentioned above. She does so in such a way that one cannot help but think that in granting her this singular form, and even more by rendering her immediately recognizable and distinguishable from the others, her inventor consciously wished to invest her with symbolic significance. At a time when a queen was called a Lady or Virgin, and *aqua fons*, it is impossible to overlook the association of lady and virgin, which occurred almost constantly in the works of Leonardo.

[32] That Leonardo was aware of discoveries of ancient Roman architectural motifs and design elements already in his youth is clear from his early works in Florence, especially the aforementioned *Adoration* and the *St. Jerome*. This was well demonstrated by Antonio Natali in LEONARDO IL GIARDINO DELLE DELIZIE, Milan: Silvana Editoriale. Natali claimed that Leonardo paid a trip to Rome. I think that even without one, the knowledge he demonstrated already in these early years he could easily have obtained from many Florentine artists who actually visited Rome in 1470-1480. Their reports and descriptions, certainly backed by drawings, found fertile soil in the young artist, and helped initiate that veritable revolution in art that Vasari called "modern" and attributed solely to Leonardo.

Next, once the proper ratio of the King's height to that of the other pieces is understood (an adjustment necessitated by the need for each piece to be inserted inside its particular square with maximum visibility), his shape becomes more elegant and his identity more immediately recognizable.

PLATE 18

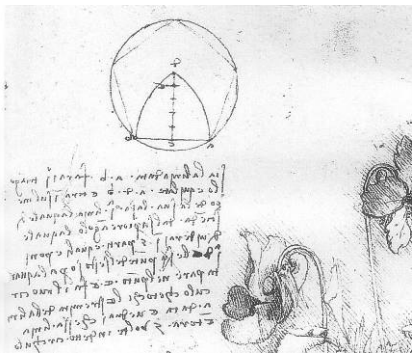


Comparison between examples of the figures of the King, Rook, Bishop, and Knight drawn from the pages of the manuscript and forms that appear in certain painted friezes on responds in the Fourth Pompeian Style.
On the right, the same forms inspire the marquetry of Giovanni da Verona at Sta. Maria in Organo, the same in which appear two reproductions of polyhedrons drawn from Leonardo's illustrations for DE DIVINA PROPORTIONE.

Finally, three-dimensionality made even more conspicuous the simple means by which the distinctions among the Rook, Bishop, and Knight were achieved. The Rook especially possesses a clear and graceful refinement, which too is indebted to a formal simplicity that cannot help but remind one of the sovereign grace of all forms created by Leonardo.

At this point, we can perhaps say that we have exhausted the proofs for verifying our hypothesis. Two reaffirmations of what has been expounded here appear in one of his plant studies (fol. 14r, Ms. B. 2173, Paris Bibliothéque National de France; pen and ink on paper, 233 x 168 mm). In the margins of a drawing of a flower with five petals, Leonardo shows how to construct a regular pentagon that has a side in common with the base of an equilateral triangle.[33] In the text next to the drawing, Leonardo crossed out two tiny errors in a left-handed manner: the first with a tiny "x", the second with two short strokes from bottom right to top left, identical to those used to delete a queen on fol. 44v and a King on fol. 1r (Pls. 19 and 12).

PLATE 19



Construction of a pentagon on the basis of an equilateral triangle; Leonardo constructs an approximate link/relationship between the hexagon, of which the equilateral triangle is one sixth, and a pentagon. The study begins with the observation of a budding five-petaled flower drawn at the side, and anticipates the correct geometric construction that binds the side of the square to the ray of the circle in the *Vitruvian Man*.

See the appended "Leonardo da Vinci: The Vitruvian Man."

Folio 14r Ms. B 2173, Paris, Bibliothèque National de France - detail

[33] The drawing is dated to between 1487 and 1490; Frank Zöllner and Johannes Nathan, LEONARDO DA VINCI: TUTTI I DIPINTI E DISEGNI, Taschen, 2003.
At this point Leonardo had not yet met Luca Pacioli, and his construction is approximate, but he intuitively anticipates the one in which he was to bond the ray of the circle to the side of the square in the *Vitruvian Man*, the geometric analysis of which I have appended to the conclusion.

The task of studying the aesthetic quality and forms of the chess pieces depicted in the manuscript attributed to Luca Pacioli was assigned to me in December 2006 by the Palazzo Coronini Cronberg Foundation O.N.L.U.S. in Gorizia in March 2007.

In May 2008 I completed the first draft of the formal analysis that constitutes a substantial portion of this study. Its conclusions, already anticipated by the press on February 18th, were sensational: the manuscript was not only – as had been authoritatively proven – the autograph and preparatory study for a work considered lost after 500 years and referred to by its author elsewhere as DE LUDO SCACHORUM or SCHIFANOIA, but possibly the result of a collaboration between Pacioli and Leonardo da Vinci.

On 3 October 2008, the Coronini Cronberg Foundation invited me to hold a conference on the subject of Leonardo da Vinci and Luca Pacioli's manuscript. The occasion granted me an opportunity to articulate the reasons underlying my conviction, and to demonstrate with new and detailed arguments that a formal analysis of the illustrated pieces would offer evidence that the manuscript, like DE DIVINA PROPORTIONE, was an intellectual collaborative project between Luca and Leonardo even though it never got beyond the stage of a work in progress.

Indeed, DE DIVINA PROPORTIONE, for which their joint authorship is indisputable, provides proofs that support my argument.

Aside from providing the historical context of the work's composition, my study singles out the stylistic elements that offer evidence of Leonardo's contribution even in the planning stage.

Both DE DIVINA PROPORTIONE and DE LUDO SCACHORUM were written in Sforza's Milan in the final years of the fifteenth century, when Luca and Leonardo were working for Duke Ludovico in a court regarded as the most splendid in Italy, and at a time when no one could have imagined its imminent and utter collapse.

All the bitterness that Leonardo felt at the drastic change imposed by this unexpected and tragic turn of events on his life is summarized in his famous comment: *"il Duca perse lo stato ella roba e libertà e nessuna sua opera si finì per lui"* [The duke lost his state, his possessions and liberty, and no work was completed for him].

What comes immediately to mind is the grand equestrian monument of Francesco Sforza, Ludovico's father. Leonardo's words, however, not only lament the incompletion of this work, which meant so much to him, but imply the disruption and end of a busy and productive existence and many assignments.

Among these may have been DE LUDO SCACHORUM, his joint project with Pacioli, which, with its mathematical implications – and not only those posed by the actual the game of chess -- was considerably more than mere play.

Today, therefore, we can imagine - perhaps not idly – that Mantua was the first stop on Luca and Leonardo's joint journey from Milan because they wished present their manuscript to Isabella D'Este.

(Luca definitely intended to dedicate the work to the Marquise - see pages 3 and 11).

In the end events forced them to depart before the treatise had assumed a form as definitive as that of DE DIVINA PROPORTIONE at the time of its presentation to Ludovico, in which Luca had had a chance to express in writing his admiration and gratitude for Leonardo's collaboration – an acknowledgment that is missing from this later manuscript only because the treatise has come down to us in a provisional draft.

The two men may have considered discussing the work with Isabella and seeking her judgment and involvement also if the job was not concluded since her immense passion for the game of chess was well known.[34]

Isabella, however, was much more interested in Leonardo the painter. Having him at her disposal at court was an occasion she had long sought. She desired a portrait of herself, and this was the task she assigned him.

[34] Isabella and the D'Este family's passion for chess, certainly fell in line with that of Borso D'Este, to whom was dedicated an illuminated manuscript on the game, now preserved in the Bibl. Reale of Turin.

The circumstantial evidence of time and place thus meshes perfectly with the conclusion I draw from the text and illustrations' style and draftsmanship.

Although my arguments were already in place before the news reached the press in February 2008, the audience was a bit skeptical as it awaited the opinion of well-known Leonardo experts. As this problem was one of confused communication, I have tried to clarify here all the issues that arose from my conclusions and to trace my course so that I could say that even on a semantic level I have done my best to fulfill the task that I had set for myself. It was with this objective in mind that I added the critical appendix, "Art and Aesthetics."

ART AND AESTHETICS

Between late December 2006, when the manuscript was attributed and February 2007, when Aldolivio Capece, editor of the journal *L'Italia Schacchistica*, introduced me to Dr. Serenella Ferrari Benedetti, coordinator of the Foundation Coronini, I had a chance to see photographs of the work's pages on the Internet.

By that point I knew that figurative representations of the game pieces had been used in place of letters or ideograms to illustrate the starting positions of the chess problems described in Pacioli's text. My interest and curiosity were piqued even if the mediocre quality of the images did not particularly excite me.

How different was my reaction to the photographs that Dr. Ferrari Benedetti brought from Gorizia!

These made a great impression on me, which certainly – though in a rather inchoate manner – gave rise to the ideas that later guided my formal analysis, in which, with the help of circumstantial evidence and stylistic comparisons, I attribute the invention of the forms to Leonardo.

The images that struck me so much then and continue to excite me today are those of the schematic chessboards with positioned game pieces. These are elegantly depicted in forms hitherto unseen, each one different though marvelously coherent with the rest.

In this section, I wish to clarify these feelings and explain my ideas through arguments that may seem irrelevant to the essay above, which remains firmly anchored in my desire to offer proofs of Leonardo's inventions. Insofar as aesthetic criticism is valid, however, I hope that by disclosing my logic I can make these proofs more persuasive.

It is necessary, for a moment, to return to the time of the manuscript's discovery and the expectations it aroused – not only among chess scholars but also among all those who, knowing the prominence and historical importance of Luca Pacioli, one of the most authoritative of Renaissance protagonists, were well aware of his association, friendship, and fruitful collaboration with Leonardo – to understand that my curiosity, though partly intuitive, was fully satisfied and has by now generated new questions, to which I admittedly cannot give immediate answers.

In particular, I have been asked about the elegance of each piece, and the logic that unifies all of them in their full lineup. Above all, I have asked myself whether a rational reply to these questions could convey with certainty that this work, like DE DIVINA PROPORTIONE, was the concrete result – beyond its undisputed paternity – of collaboration between Luca and Leonardo.

Starting out with feelings, one goes beyond reason and proofs to recognize – with Benedetto Croce – that these alone convince us of the presence of a work of art.

And in this case a very particular one work of art. As this chess set didn't consist of figurative, decorative and representational pieces, that should allow us to appreciate the artist's hand direct intervention in the execution of each piece, but it is rather comparable with a modern work of industrial art.

In this particular case the maker, obviously, did not intervene directly however, beyond his work, we can appreciate and recognize the sensibility and creative power that determined the creation of each piece as well as in their mutual relationships.

Today we would call such a work "design".

When the result of “design” is a work of art, it is easy to recognize the period in which it was produced and occasionally even its actual maker. In addition, we know that industrial execution and endless reproducibility are no obstacle to quality; an industrial product may also be a work of art and appropriate tools of logic may be used for discovering its authorship.

Naturally the criteria with which such works are analyzed differ from those used for a painting, a sculpture, or more generally a unique work of art in which it is fundamental to recognize not only an idea but also the artist's hand.

Alois Riegl was the first to liken industrial art to architecture; in SPÄTROMISCHE KUNSTINDUSTRIE, he tried to formulate rational tools for evaluating the aesthetic quality of late Roman industrial art and thus to identify more easily its various styles. Indeed, industrial art and architecture are united by the fact that they have no iconographical value, which is innate to works of sculpture and painting, at least until Riegl's time, that is, before the emergence of non-figurative art.

My stylistic analysis of the unprecedented forms of these chess pieces reveals a consonance in their proportions as well as a relationship between them and certain known architectural elements designed by Leonardo, even though these can be found in only a few elements of architectural structures attributed to him.[35]

At the same time, however, any attempt to uncover the significant elements of these geometric and abstract formal motifs in representational paintings of a real or imaginary world is bound to fail.

In any case industrial art, though lacking in or even void of historical or iconographic significance, is nonetheless art thanks to the characteristic and essential coherence of its function and the means of its production, and in that it expresses emotion because it makes us interested in the sentiments of its maker.

As Croce notes, it offers a window into its world -- immediate, that is, unmediated, and direct communication. Only at a second instance can its language be analyzed to uncover and understand its underlying rules and recognize its style, which alone can reveal its period, its place, and occasionally even its maker.

For this reason my study is aimed above all at underscoring the work from this point of view.

Different, perhaps innumerable, are the languages we use to communicate. Nevertheless all stem from the two primary and fundamental ones from which they evolved: primordial and instinctual, that is, corporeal and gestural communication: the spoken word, sound, later song and music, and drawing, modeling, and formal design -- and the infinite number of derivations and combinations of these two modes.

An artist may realize a work of art in any of these languages.

Nature desires that over time some acquire the sensibility, culture, and tools to express themselves at the highest level, which we recognize as art.

Natural talents lie hidden and endow certain people with every kind of ability. In order that these talents be thoroughly realized and returned to the world through works created by those who have been granted them, there must be an environment suitable for their evolution.

This was as true for Giotto, Mozart, or Leopardi as for Leonardo.

In the indisputable precociousness of Leonardo, in his natural gifts for capturing nature with great fidelity to nature, the precision of his line, his use of color and *sfumato*, and the perfection of his proportions and perspective, the analysis of which fills so many scholarly volumes – I have always been fascinated by two particular life-long aspects of his personality: his elegance, referred to by Vasari, Bellincioni and Pacioli, and his generosity, that is, his readiness to engage with other artists close to him.

Without any deprecation, we can define Leonardo’s elegance as light, the essence of a serene, non-conflicting beauty -- not an insipid, vacuous lightness, but a serious, peaceful and self-conscious one that only an innocent child is capable of expressing. For example, regarding Leonardo's female portraits, his Madonnas and his angels, we can see that they are all beautiful, serene, and similar in appearance. They are bound by what I have termed "serene beauty," one touched by the celestial and supernatural. Whenever we encounter the same smile and similar features in a face painted by any other artist of the period, we ask ourselves

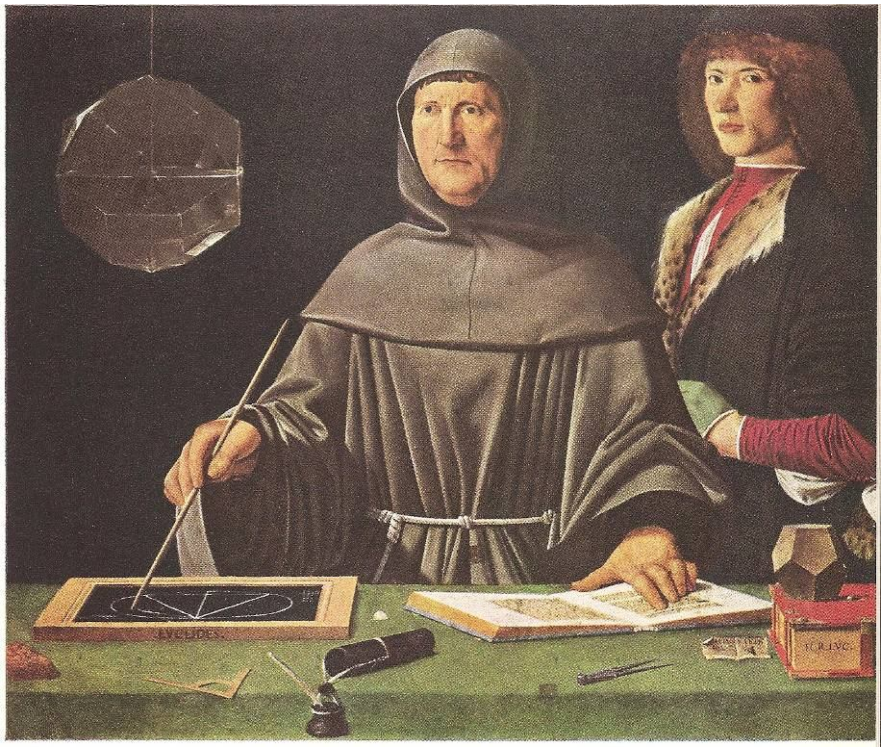
[35] C. Pedretti, LEONARDO ARCHITETTO, Milan: Mondadori Electa Spa, 2007, pp. 79-80.

whether Leonardo had had a hand in the work, and if not he, then at least his workshop.

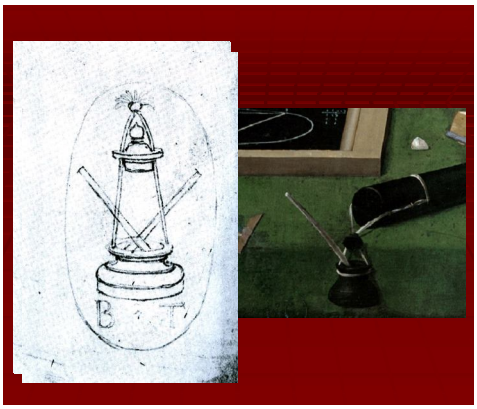
Finally the natural, serene elegance of Leonardo is not only a characteristic of the faces he painted but also of himself, of his nature, of his manner of dressing, and of all the things with which he surrounded himself. It was his trait, in short, his style, and it is this that I see in the chess pieces of Pacioli's manuscript. These, in fact, are reduced to essentials, perfectly adapted to the function and stripped of superfluous ornament.

If we wish to find a reference to this in Leonardo's other works, we must look at the essential quality of the machines and instruments that he invented and designed, at their functionality, and not at the decoration of the bookrest grazing the hand of the Madonna in his splendid early *Annunciation*, or the ornaments designed for a helmet or a sword hilt, since in these he was not thinking about or wishing to communicate the concept of form but was rather creating ornament for its own sake and as a quality of a particular period and social condition. More closely related to chess pieces in terms of dimension and functionality is the inkstand and pen holder, an "emblem," a sort of ex libris, which Leonardo designed for a friend, and which appears in the famous portrait of Luca Pacioli, attributed to Jacopo de Barbari, now in the Museo Capodimonte in Naples (Pl. 20).

PLATE 20.



Portrait of Luca di Pacioli traditionally attributed to Jacopo de Barbari,
Museo Nazionale di Capodimonte, Naples



Detail of the inkstand with pen case.

At its side, the same inkstand depicted in a drawing by Leonardo, ca 1500, Codex Atlanticus, fol. 306 r-a, supposedly an "emblem" meant for a friend, Tovaglia or more probably Bartolomeo Turco.
According to C. Pedretti, LEONARDO ARCHITETTO, p. 308.

Notable, stylistically speaking, is the greater elegance and gracefulness of the inkstand drawn by Leonardo vis à vis the object depicted in the portrait.

In it too – in its simple adherence to function -- there is no extraneous ornament. As in the case of the chess pieces, had the evolution of writing

techniques not relegated the inkstand to the world of antiquarians, we could define it too as modern, just as we do these chess pieces.

Stylistic affinities likewise exist between the form of a fountain and the queen in DE LUDO, and especially, as I have demonstrated, in the use of the Golden Ratio in the geometric configuration of the pawn.

This interest in geometry corresponds to a specific period of Leonardo's life, and these examples with several others constitute an exceptional case, since the artist's natural instinct when drawing, painting, and modeling supernatural -- so-to-speak -- beauty was never organized by geometric rules. In my study, I have demonstrated how those works of his in which such rules are detectable in figures and compositions all fall within one brief period that extends from the polyhedrons of DE DIVINA PROPORTIONE to the chess pieces of DE LUDO.

In fact, as we well know, Leonardo was absorbed throughout his life in the study of the rules of harmony in nature. Only in this one brief period did he use the Golden Ratio to achieve proportion in his works. The awareness he acquired was sufficiently profound to make him realize that the complexity of natural phenomena could not be reduced to a geometric formula that would serve as the framework for his art.

Although other distinguished artists, such as Piero della Francesca, had applied the rule to their works regularly throughout their lives, Leonardo examined and delved into every aspect of it during only a few – three or four – years prior to DE DIVINA PROPORTIONE. Later, in the *Vitruvian Man*, he drew from it the essential bond to which even man must submit. Finally, in the *Last Supper* he used it as the basis for his compositional structure.

It may be his grace – a distinct and prominent feature of Leonardo's style, on a par with his generosity – that explains the reason why he never mentions any sort of collaboration on DE LUDO with Pacioli in any of his writings.

Although direct proof of this is unavailable, it is possible to ascertain his instrumental contribution indirectly through other works by third parties, especially DE DIVINA PROPORTIONE. Not a single trace of a preparatory drawing for any of the sixty drawings illustrating its text appears in the thousands of pages of Leonardo's surviving notebooks. At the same time, considering their originality and complexity, we cannot presume that they rose to their definitive state out of nothing. Moreover, it would be too reductive to assume that Leonardo's collaboration was a critical one and limited to translating through graphic means what his friend Pacioli was laboriously describing. So much so that it seems useless for me to dwell on how innovatively DE DIVINA examines five regular polyhedrons, and how clearly Leonardo's drawings illustrate their every possible harmonic evolution. More than illustrating the text, these have left innumerable traces on the works of his contemporaries and successive generations.

In the same way, without wishing to rewrite here the history of his life, I note Leonardo's collaboration in resolving the problem of the dome-cladding of Milan's Duomo, for which there remains little evidence, as well as his many studies of the centrally-planned church. Although there is no information as to whether these materialized into actual works, the impact of his designs appear in the works of many architects of his time, and in particularly in that of Bramante – a collaboration for which too there is no substantiation save their friendship and association in Ludovico's Milan, when they worked at close quarters in the church of Santa Maria delle Grazie. One was absorbed in his masterpiece, the *Last Supper*, the other – still far from the mature artist that he would become in Rome with the Tempietto in Montorio – in designing a centrally-planned structure.

Who could doubt which of the two was the one to give without receiving anything and which was the one who received without offering anything but friendship in return?

Perhaps this is intimated in the fresco of Democritus/Bramante, who laughs, and Heraclitus/Leonardo, who cries, or perhaps more correctly (to anyone who looks carefully) who too laughs, but with tears in his eyes (Pl. 21).

Did not Carlo Pedretti see in this painting by Bramante Leonardo's hand, and possibly his discreet or involuntary signature in the left-handed writing style of the text of the book handled by Democritus/ Bramante?[36] Thus discreet collaboration was nothing new for Leonardo and no cause for wonder.

[36] See p. 97 in LEONARDO ARCHITETTO, op. cit, n. 35 above.

PLATE 21.

Donato Bramante, *Heraclitus and Democritus*, Biblioteca di Brera, Milan
In LEONARDO ARCHITETTO, p. 98, Pedretti advances the hypothesis that Leonardo worked on this fresco. In particular, the foreshortening of Democritus' raised hand, the refined orography of the terrestrial globe suspended between the two philosophers, and the left-handed mirror-writing on the open books before Heraclitus, point straight at Leonardo.
Other considerations lead to the assumption that behind the likenesses of these two philosophers lie representations of Bramante in the garb of Democritus and Leonardo in that of Heraclitus.
Though probable, the collaboration was not obvious, as was nearly always the case with those friends and artists with whom Leonardo was in contact. This was Leonardo's style. Typical of this is Antonio Natali's recent discovery of the artist's hand in the rendering of Christ's face in Verrocchio's *Baptism of Christ*. See Pl. 22.

In my essay I recall Magni-Duffloc's attribution of the invention of the violin to Leonardo. No one asked Leonardo for such a thing, and as in the case of his other extraordinary inventions, there is no documentation that he contributed to this one. All the same, Leonardo's definite participation in or contribution to projects is undocumented in so many cases that a book could be written on the subject.

Perhaps the earliest instance of this lies in his contribution to Verrocchio's *Baptism*, which has only been recognized very recently, but which for this reason is all the more important for understanding his personality and style (Pl. 22).

Everyone agrees on the grace and beauty of the angel in the foreground, painted by the not more than nineteen-year-old Leonardo. Critical analysis has always dwelt on the contrast between his features and the hard physiognomy of the faces and bodies of Verrocchio's figures.

This was a contrast so obvious that it spawned a legend -- nurtured by Vasari -- that the master was so disconcerted by the aptitude of his young pupil that he refused to paint angels forever after.

The legend is irrelevant here; I dwell merely on the fact that up until recently critical analysis has discerned a change in the work of the older master in response to that of Leonardo. Recent photographic analyses, however, have found evidence of the impression of Leonardo's finger on the face of the baptized Christ. Thus it seems that the young Leonardo went beyond the task assigned him, and with a daub of oil paint – that we may imagine as swift and light – intervened over the one just applied by his master in order to soften the image of Christ's face. Perhaps it was in completing this gesture that he leaned over the angel and left an indelible impression of the weave of his garment where the painting was still drying.[37] This gesture betokens the spontaneity of one who is conscious of his own ability, but dispenses it with generosity.

[37] Antonio Natali, LEONARDO: IL GIARDINO DELLE DELIZIE, Milan: Silvana Editoriale, 2002.

Leonardo gave away his works without signing them. He never boasted of what he was going to do; on the contrary, we read in his writings only his regret at not bringing something to a successful conclusion.

Why then not search for traces of his authorship of chess pieces in a work that Pacioli meant to complete in a manuscript?

Luca had been collecting chess problems and notes about the game for a while. He wished to write a tract on the game of chess. As he organized and recorded the moves that he described, he certainly had at his disposal game pieces on a chessboard.

Leonardo could not have been ignorant of this or not have seen it. The two were friends; together they confronted complex problems of geometry and worked out their philosophical implications.

PLATE 22.

Andrea del Verrocchio, Leonardo da Vinci, et al. *The Baptism of Christ*, Galleria degli Uffizi, Florence

For a complete discussion of the known and recently discovered contributions of Leonardo to this work, see Antonio Natali, LEONARDO. IL GIARDINO DELLE DELIZIE, Cinisello Balsamo, Milan: Silvana Editore, 2002.

Leonardo's solution for these was always graphic, immediate, and supremely elegant. So much so that Luca noted "his divine left hand..." Why not hold as more than likely that he also intervened in the game – which had always been an exercise of intelligence – with the same ease for finding solutions and suggesting new problems?

Finally, what about the pieces that move on the surface of the chessboard those that they studied and with which they played? One look at those used at the time is enough to comprehend that Leonardo had arranged his own on the chessboard, those that he himself had conceived and modeled. Is there, however, any trace of these in his studies, in his notes?

Of the notes, which Pacioli had and about which he wrote *"idem habes in meis quinternis,"* Leonardo had no need. As for the game pieces, he certainly did not have to make them. He simply modeled them.

Beneath his fingers developed a new form, a wonderful synthesis of geometric rules that he was discussing with Pacioli at precisely this time and the elegance of Roman ornaments that were being discovered and admired in these very years at all the courts of Italy. And Leonardo – from his earliest documented works, such as the *Adoration of the Magi*, in which he used and transformed many motifs from rediscovered Roman antiquities on view at the court of Lorenzo the Magnificent – was endowed with a gift for re-elaborating these ornaments.

Thus circumstantial evidence of time and place, proofs related to the clearly expressed objective to proceed with the industrial manufacturing of the pieces and the formulation of the materials to be used, proofs based on the style, on the examination of the idea itself as well as the archetypical model, and last but not least, proofs drawn from a graphic analysis, recognize the authenticity of the master's authorship.

These, each and all together, are the reasons underlying my conviction.

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